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Bertrand Competition in Markets with Network Effects and Switching Costs*

Irina Suleymanova† Christian Wey‡

May 2008

Abstract

We analyze market dynamics under Bertrand duopoly competition in industries with network effects and consumer switching costs. Consumers form installed bases, repeatedly buy the products, and differ with respect to their switching costs. Depending on the ratio of switching costs to network effects, our model generates convergence to monopoly as well as market sharing as equilibrium outcomes. Convergence can be monotone or alternating in both scenarios. A critical mass effect, where consumers are trapped into one technology for sure only occurs for intermediate values of switching costs, whereas for large switching costs market sharing is the unique equilibrium and for small switching costs both monopoly and market sharing equilibria emerge. We also analyze stationary and stable equilibria, where we show that a monopoly outcome is almost inevitable, if switching costs or network effects increase over time. Finally, we examine firms' incentives to make their products compatible and to create additional switching costs.

JEL Classification: L13, D43, L41

Keywords: Network Effects, Switching Costs, Bertrand Competition, Market Dynamics

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1 Introduction

Competition in many parts of modern economies, and in particular, in so-called high tech industries is increasingly characterized by technologies which give rise to pronounced network effects and by switching costs consumers have to forego when they change the technology (for recent surveys, see Klemperer, 2005, and Farrell and Klemperer, 2007).[1] Technologies are typically either completely or at least partially incompatible.[2] Though products may be differentiated as usual, its importance for consumers' purchasing decisions is often negligible when compared with their preference for *compatible* products.[3] Both switching costs and network effects have attracted concerns in competition policy circles about the effectiveness of competition (see, e.g., FTC, 1996, and OECD, 1997).[4] While switching cost have been alleged to ease the competitive pressure among firms, network effects have raised concerns that persistent monopolies are inevitable. Both market forces have been studied intensively, though virtually the entire literature focused on one of the forces exclusively (we present the relevant literature below). It is, therefore, fair to say that little is known about the interplay of switching costs and network effects which we believe is rather the norm than the exception in real world markets. This paper aims at closing this research gap.

We observe strikingly different market dynamics when incompatible technologies compete against each other and *both* network effects and switching costs are essential features of the market. In many instances, competition between technologies leads to a persistent monopoly outcome where one technology becomes the de facto standard and rival technologies are com-

[1] The competitive forces in markets with network effects and switching costs have been described in an increasing number of business and market studies; see, for instance, Grindley (1995), Shapiro and Varian (1998), Rohlfs (2001), and Gawer and Cusumano (2002).

[2] Incompatibilities are the norm when firms start to market new products and technologies are protected by business secrets and/or property rights (patents or copyrights).

[3] Not surprisingly, there are numerous stories about alleged "market failures" when consumers have a desire for compatibility. To mention some examples, the QWERTY keyboard standard, Microsoft's operating system MS DOS, or the videocassette recorder standard VHS have all been proscribed as inferior to their losing rivals, namely, Dvorak (see David, 1985, and Liebowitz and Margolis, 1990 for an opposing view), Apple (see, e.g., Shapiro and Varian, 1998), and Beta (see Cusumano, Mylonadis, and Rosenbloom, 1992), respectively.

[4] Policy implications are also discussed in the surveys of Klemperer (1995), Gandal (2002), and Farrell and Klemperer (2007).

pletely driven off the market. In other instances, market sharing outcomes prevail such that incompatible standards compete head-to-head. Another characteristic feature of those markets is that the evolution of market shares is sometimes rather monotone while in other instances market dominance alternates over time. If convergence towards monopolization is monotone, then a dominant firm expands its market share from period to period to the point of complete monopolization. Under monotone convergence towards a market sharing outcome, an initially dominant firm remains dominant but loses market shares to the rival firm which expands its market share over time accordingly. In contrast, under alternating dynamics, dominance changes over time. Again, alternating dynamics may either develop towards complete monopolization or towards a market sharing outcome. Moreover, markets with network effects often exhibit a so-called "critical mass" effect such that the firm which reaches the critical mass of users at first completely monopolizes the market thereafter.[5]

A famous case of a monotone monopolization process is the QWERTY keyboard standard (see David, 1985, and Arthur, 1989). The market for compact disks and CD players provides another example, where the standard introduced by Phillips and Sony in 1983 rapidly became the de facto standard in the industry (see McGahan, 1991a/b for a description of this case). Monopolization was also the outcome in the VCR standards battle between VHS (sponsored by JVC) and Beta (sponsored by Sony). However, dominance dramatically alternated in that case: While Beta benefited from a first-mover advantage and obtained a dominant position in the early seventies, VHS managed to displace Beta completely after a period of more than ten years.[6] Similarly, market dominance altered in the early years of the famous rivalry between Apple's and Microsoft's operating systems. Another example illustrative for alternating dominance was the competition between AM and FM standards in radio broadcasting (for a detailed description of this case, see Besen, 1992). Consumers were initially reluctant to buy FM receivers since they had to bear switching costs and were uncertain about the other users' propensity to switch. Thus it took about thirty years for the FM standard to get more than fifty percent of the market although it was considered to be a superior broadcasting standard. This case also highlights

[5] See Rohlfs (1974), Arthur (1989), and Shapiro and Varian (1998) for the role of the "critical mass" in markets with network effects.

[6] The standards war between Beta and VHS is extensively described in Cusumano, Mylonadis, and Rosenbloom (1992) which features also the evolution of market shares.

the role of the critical mass effect which marks the extinction of the rival technology. Recently, Toshiba decided to pull out of the HD DVD business so that the rival format Blu-ray sponsored by Sony is expected to dominate that market in the near future.[7] The decision was announced by Toshiba just after Time Warner (a worldwide leading movie producer) decided to support exclusively Blu-ray. As Toshiba held a larger installed base than Sony at the time of announcing the withdrawal of its technology from the market, the associated market dynamics mirror an alternating process towards complete monopolization.

A striking market sharing outcome between (partially) incompatible standards is documented in Augereau, Greenstein, and Rysman (2006) who study the adoption of 56K modems by internet service providers in the US in the late nineties. Similarly, the market for videogame concoles is shared between three major producers (namely, Nintendo, Sony, and more recently, Microsoft). Dominance has alternated in the videogame industry. Nintendo held a dominant position in the eighties and nineties, then lost its dominance while, most recently, it appears to have strengthened its market position relative to its rivals.[8] Another example for an alternating market sharing outcome can be seen in the coexistence of different standards in wireless telephone networks (namely, CDMA, TDMA and GSM) in the US (see, Gandal and Salant, 2003).

A closer investigation of all those cases, of course, may give rise to many explanations for the particular market dynamics under specific market environments. However, at a more general level, all those cases share some common features: *Firstly*, few (in most cases only two) incompatible technologies compete against each other; *secondly*, network effects play an important role in determining the value of a technology; and *thirdly*, consumers have to bear switching costs if they decide to substitute one technology against the other.[9]

In this paper we develop a model of duopolistic competition between incompatible technologies that incorporates *both* network effects and switching costs. Our main contribution is to analyze how the interplay between both market forces shapes competitive outcomes and market

[7] See "Toshiba is Set to Cede DVD-format Fight," Wall Street Journal Europe, February 18, 2008, p. 3.

[8] See "Wii and DS Turn Also-Run Nintendo Into Winner in Videogame Business," Wall Street Journal online, April 19, 2007 (http://online.wsj.com).

[9] Moreover, consumers do switch technologies in equilibrium; a phenomenon absent in most of the existing literature (as we will see below in the literature review).

dynamics. Within a single model we can show that the emergence of the above described dynamics critically depends on the ratio of switching costs to network effects. More precisely, we consider a single cohort of consumers who repeatedly buy the products which only differ with regard to network effects and switching costs. Initially, all consumers are allocated to either of the firms' installed bases. Firms' products are incompatible and each technology gives rise to proprietary network effects which are linearly increasing in the number of users. Consumers have to bear switching costs if they switch the technology. Switching costs increase symmetrically and linearly over the set of users of each technology. Firms compete in prices and we search for Bertrand equilibria where consumers hold rational expectations which are fulfilled in equilibrium. The analysis of our model reveals that market dynamics then critically depend on firms' installed bases and a single parameter which measures the relative importance of switching costs compared to the intensity of network effects. For the considered parameter space we obtain all relevant cases. When switching costs dominate network effects, then a monotone convergence to the market sharing outcome follows (as a unique equilibrium outcome), while in the opposite case (i.e., when network effects dominate switching costs) multiple equilibria follow with market sharing and monopolization as possible outcomes. In that area the dynamics in the interior solution (i.e., the market sharing equilibrium) is strikingly different from the case, when switching costs dominate network effects. While in the latter case convergence towards market sharing is monotone, in the former case convergence follows an alternating path.

We also identify an intermediate range of parameters where network effects and switching costs are more balanced. In that region market dynamics critically depend on the size of firms' installed bases. Moreover, the market dynamics are markedly different from the previous cases. If a market sharing equilibrium exists, then it always converges towards monopolization. Interestingly, convergence can be either monotone or alternating. In the latter case dominance alters, such that the new dominant firm obtains a larger market share at the end of the period when compared with the market share of the initially dominant firm. Moreover, there exists also a region where a critical mass effect occurs, such that the initially dominant firm becomes the monopolist for sure (i.e., as a result of a unique equilibrium outcome) at the end of the period. Both patterns are absent when either network effects or switching costs dominate each other. Our analysis, therefore, reveals that the interplay between switching costs and network effects gives rise to new results, absent in previous works that focused on either one of both market forces. Moreover, we also analyze how the type of equilibrium (either market sharing or monopolization) affects consumer surplus and social welfare, where we show that a fundamental conflict arises between both welfare goals. While positive network effects require consumers to coordinate on one particular technology, consumer surplus is generally higher when both firms compete head-to-head.

We consider several extensions of our basic market model. *First*, we derive stable and stationary equilibria if the market game is played infinitely often, where we abstract from issues of intertemporal optimization (i.e., we suppose that all agents behave myopically). *Second*, we investigate how the longer run equilibrium outcome is affected if switching costs or network effects increase over time. We show that complete monopolization by either one of the firms then becomes highly likely. *Third*, we analyze firms' preferences for making their products compatible and we examine firms' incentives to increase switching costs.

Our paper contributes to the literature that deals with imperfect competition in markets with network effects and switching costs. There is a large literature on both market forces, however, besides few exceptions (in particular, Farrell and Shapiro, 1988), the literature focuses mainly either on network effects or switching costs exclusively.[10] With regard to network effects, our paper builds on the seminal paper by Katz and Shapiro (1985) which incorporates network effects into the Cournot oligopoly model. We adopt their concept of a fulfilled expectations equilibrium to our model of Bertrand competition. Katz and Shapiro obtain multiple equilibria for the case of incompatible products. Precisely, they show existence of a symmetric equilibrium where firms share the market equally as well as asymmetric equilibria, where the market becomes more concentrated. We obtain qualitatively similar results, whenever network effects dominate switching costs. However, we also consider installed base effects (which are absent in Katz and Shapiro, 1985, who only consider symmetric firms), which allows us to investigate market dynamics in a market sharing equilibrium.

The dynamics of markets with network effects has attracted a lot of attention in the litera-

[10] As we focus in our literature review on those contributions most closely related to our model we do not touch on important related issues, as e.g., price discrimination or price commitments that are not part of our analysis (again, we refer to the excellent survey by Farrell and Klemperer, 2007).

ture. Those works focused on markets where consumers enter sequentially and make irreversible adoption decisions. Intertemporal network effects and consumer lock-in typically lead to a monopolization outcome and several dynamic inefficiencies; most notably, excess inertia and excess momentum (see, Farrell and Saloner, 1986, Katz and Shapiro, 1986, and Arthur, 1989). The dynamics are mainly driven by asymmetries between technologies (in particular, in the form of product differentiation, technological progress, and different times of arrival in the market place). In contrast, in our model firms' products are inherently symmetric (i.e., in terms of their network-independent utility, production costs, and arrival date), but may differ with respect to their installed base. Moreover, Farrell and Saloner (1986) as well as Arthur (1989) only analyze consumers' adoption decisions while product supply is perfectly competitive. Duopolistic price competition in a two-stage model where different consumer cohorts enter sequentially and intertemporal network externalities occur, has been analyzed in Katz and Shapiro (1986). Again, that model assumes perfect consumer lock-in, so that switching incentives are not analyzed.

Klemperer (1987a/b) are seminal contributions to the switching costs literature that examine (besides many other things) the "bargains-then-ripoffs" incentives in a two-stage market environment with consumer switching costs. Switching costs tend to reduce competition, and thereby, may also benefit firms to the expense of consumers. In a dynamic setting with overlapping consumer generations, a fat-cat effect results from switching costs (modelled as perfect consumer lock-in) which creates an entry-inducing effect. That effect has also been analyzed in Farrell and Shapiro (1988), where it is also shown that the result is robust vis-à-vis (not too large) network effects. Their model gives rise to rather extreme dynamics where the entering cohort of consumers always buys from the entrant firm.[11]

Dynamic duopoly competition in markets with switching costs was analyzed in Beggs and Klemperer (1992). They consider a dynamic model with "new" and "old" consumers and differentiated products. Firms and consumers are forward looking and consumers face prohibitive switching costs. It is shown that market shares converge monotonically to market sharing in a Markov perfect equilibrium. The larger firm sets a higher price than its competitor to exploit its consumer base, and thereby, attracts less new consumers, and thus, loses its dominance over

[11] As we will show below, such an extreme type of alternating dominance (where firms interchange market shares) is also an equilibrium outcome in our model which occurs for a particular parameter constellation.

time. To (1996) analyzes a model very similar to the one of Beggs and Klemperer (1992) with the only difference that consumers live for just two periods. He shows existence of a unique Markov perfect equilibrium, where firms' market shares converge in an alternating fashion; a result similar to the one obtained in Farrell and Saloner (1988). Let us reiterate that the cited literature analyzes a growing market where consumers are perfectly locked-in after their first purchasing decision.[12] In contrast, we focus on market dynamics when consumers can switch technologies so that competition among firms is shaped by consumer switching costs as well as by network effects.

Our paper proceeds as follows. In Section 2 we present our basic market model and in Section 3 we derive and characterize equilibrium outcomes. In Section 4 we consider three extensions: *firstly*, we examine the dynamic extension of our market game and analyze stable and stationary equilibria, *secondly*, we analyze firms' incentives to make their products compatible, and *thirdly*, we investigate firms' incentives to increase or mitigate switching costs. Finally, Section 5 concludes.

2 The Model

We consider two firms, $i = A, B$, that produce incompatible products A and B, respectively. We normalize production costs to zero. Firms compete in prices, p_i ($i = A, B$), which they determine simultaneously. Given p_A and p_B, consumers make their purchase decisions. All consumers have the same valuation of the stand-alone value, $v > 0$, of the products which we assume to be sufficiently high such that the market is always covered. The consumption of a product creates positive network effects for users of the same product. We suppose that consumers' utility is linearly increasing in network size, with each additional consumer creating a constant positive externality, $b > 0$, to the utility of the users of the same product.

We assume a continuum of consumers with a mass of one. We suppose that at the beginning of the period each consumer belongs either to the installed base of firm A or B.[13] Hence,

[12] A notable exception is Caminal and Matutes' (1990) analysis of loyalty discounts.

[13] Emerging markets for network goods typically develop rather randomly in their very early stages. Overall uncertainty in the market is large and small events (David, 1985, and Arthur, 1989) may induce consumers to decide for one of the products without foreseeing the implications entirely. An exogenous installed base may also

before price competition occurs, each firm already holds an exogenously given initial market share (the so-called installed base), $\alpha_i^0 \in [0,1]$, with $i = A, B$. As we assume that the market is always covered, market shares must add up to unity; i.e., $\alpha_A^0 + \alpha_B^0 = 1$. While in the beginning of the period each consumer belongs to either of the installed bases of the firms, every consumer can switch to the other firm's product. However, switching is costly, whereas buying the prior technology again does not create similar costs.[14] Consumers of each installed base are differentiated with respect to their switching costs. We require that switching costs for any distribution of installed bases fulfill the following properties: *Firstly*, there is always a consumer of infinitesimal size with zero switching costs, and *secondly*, switching costs increase symmetrically and linearly over both installed bases.[15]

Precisely, let consumers be uniformly distributed on the unit interval such that each consumer obtains an address $x \in [0,1]$. We suppose that the installed base of firm A lies in the interval between $x = 0$ and $x = \alpha_A^0$ and the installed base of firm B lies in the remaining part of the unit interval, i.e., between $x = \alpha_A^0$ and $x = 1$. Applying both requirements, we can then write the switching costs for a consumer with address x as $t\left|\alpha_A^0 - x\right|$, where $t > 0$ is the slope of the switching cost function.[16] Our specification of consumer switching costs is a natural extension of the well-known Hotelling model of horizontal product differentiation into a setting where installed bases determine switching costs and, with that, product differentiation.

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be the result of several promotional activities (e.g., targeted sales or free test products) of the firms.

[14] There are many reasons for consumer switching costs as, for example, technology-specific learning effects or sunk investments into complementary equipment which is incompatible with other brands (see Klemperer, 1995, for a comprehensive list of the many sources of consumer switching costs).

[15] The first assumption avoids discontinuities and the second assumption assures that firms' optimization problems remain symmetric (in the interior solution) besides possibly unequal installed bases. See also Klemperer (1987a) for a discussion of different specifications of consumer switching costs.

[16] The slope of the switching cost function may change with the size of a firm's installed base. If we, for instance, assume that consumers' switching costs are uniformly distributed over a certain interval independently of a firm's installed base, then switching costs increase more rapidly over the set of users of the firm which holds the smaller installed base. It is easily checked that such a specification would make a monopolization outcome less likely, while not affecting our results qualitatively. Moreover, the opposite is also conceivable, as peer-effects which help new customers to join the network may increase the larger a firm's installed base becomes (Henkel and Block, 2006). Being agnostic about the exact relationship between a firm's installed base and the shape of the switching cost function, we require symmetry which simplifies our analysis in the most convenient way.

We denote firms' market shares at the end of the period by α_i^1, with $i = A, B$. As switching is costly, the utility of a consumer located at the point x depends on its initial allocation to one of the installed bases and its switching costs. The utility of consumer x from buying product i can then be written as[17]

$$U_x^i = \begin{cases} v + b\alpha_i^1 - p_i & \text{if} \quad x \in \alpha_i^0 \\ v + b\alpha_i^1 - p_i - t\left|\alpha_A^0 - x\right| & \text{if} \quad x \in \alpha_j^0, \end{cases}$$

for $i, j = A, B$ and $i \neq j$. Thus the utility of a consumer who is loyal and stays with product i, is the sum of the stand alone value of the product, v, and the overall network utility, $b\alpha_i^1$, minus the product price, p_i, while a consumer x who switches technologies has to bear additional switching costs, $t\left|\alpha_A^0 - x\right|$. Firm i's new market share at the end of the period, α_i^1, may differ from its initial installed base, α_i^0, if consumers switch.

The timing of the market game is as follows: First, consumers form expectations about firms' market shares and firms set prices, p_i, with $i = A, B$, simultaneously so as to maximize their profits which are given by $\pi_i = \alpha_i^1 p_i$. Then, consumers observe firms' prices and make their purchase decisions.

3 Equilibrium Analysis and Main Results

We search for fulfilled expectations Bertrand equilibria in which every firm sets its price given the price of the competitor and consumers' expectations about future market shares which we denote by α_i^e for $i = A, B$.[18] In a fulfilled expectations Bertrand equilibrium, firms' equilibrium market shares, α_i^*, equal expected market shares, such that $\alpha_i^* = \alpha_i^e$ must hold, while firms' equilibrium prices p_A^* and p_B^* are determined by

$$p_i^* = \arg\max_{p_i} \pi_i(p_i, p_j^*, \alpha_i^*; \alpha_i^0) \text{ for } i = A, B \text{ and } i \neq j.$$

To find the equilibrium we will proceed backwards and start with consumers' purchase and switching decisions for given prices and given expectations. This yields firms' demand func-

[17] With some abuse of notation let α_i^0 also denote the set of consumers on the unit interval which forms the installed base of firm i ($i = A, B$); i.e., $\alpha_A^0 = \{x | 0 \leq x \leq \alpha_A^0\}$ and $\alpha_B^0 = \{x | \alpha_A^0 \leq x \leq 1\}$.

[18] The concept of a fulfilled expectations Bertrand equilibrium is borrowed from Katz and Shapiro (1985) with the only difference that in our case firms compete in prices and not in quantities.

tions. We then solve for firms' optimal prices for given expectations. Finally, we require that equilibrium market shares are equal to consumers' expectations.

For given expectations and prices every consumer chooses the product which provides him the highest utility. As v is assumed to be sufficiently large so that the market is always covered, all consumers ($x \in [0,1]$) for whom $U_x^i \geq U_x^j$ holds choose product i with $i,j = A,B$ and $i \neq j$. Setting $U_x^A = U_x^B$ and solving for the marginal consumer who is indifferent between the products of the two firms, yields

$$\alpha_A^1 = \min\{\max\{0, \alpha_A^0 + [p_B - p_A + b(2\alpha_A^e - 1)]/t\}, 1\}.$$

We can now express the demands for the firms' products for given expectations, prices, and installed bases as

$$\alpha_i^1(p_i, p_j, \alpha_i^e; \alpha_i^0) =$$
$$\begin{cases} 0 & \text{if} \quad p_j - p_i \leq -t\alpha_i^0 - b(2\alpha_i^e - 1) \\ \alpha_i^0 + \frac{p_j - p_i + b(2\alpha_i^e - 1)}{t} & \text{if} \quad -t\alpha_i^0 - b(2\alpha_i^e - 1) < p_j - p_i < t(1-\alpha_i^0) - b(2\alpha_i^e - 1) \\ 1 & \text{if} \quad p_j - p_i \geq t(1-\alpha_i^0) - b(2\alpha_i^e - 1), \end{cases} \tag{1}$$

with $i,j = A,B$ and $i \neq j$. We have to consider two types of equilibria: *First*, an interior equilibrium in which both firms serve the market with strictly positive market shares, and *second*, corner solutions where one firm monopolizes the entire market. We refer to the former equilibrium as the "market sharing outcome" and to the latter equilibrium as the "monopoly outcome."

Before proceeding with the analysis, let us define the ratio of switching costs to network effects by $k := t/b$, with $k \in (0, \infty)$. The new parameter, k, measures how important network effects are relative to switching costs. For relatively small values of k, network effects (switching costs) are more (less) important than switching costs (network effects), whereas for relatively large values of k, the opposite holds. We start with the equilibrium analysis of the market sharing outcome.

Market sharing outcome. In an interior equilibrium firms' first order conditions must be fulfilled for market shares that lie within the unit interval and nonnegative equilibrium prices. According to (1) the demand for firm i in an interior equilibrium is given by

$$\alpha_i^1(p_i, p_j, \alpha_i^e; \alpha_i^0) = \alpha_i^0 + \frac{p_j - p_i + b(2\alpha_i^e - 1)}{t} \text{ for } i = A,B \text{ and } i \neq j. \tag{2}$$

Maximizing $\pi_i(p_i, p_j, \alpha_i^e; \alpha_i^0)$ with respect to p_i we obtain firm i's first order condition

$$\alpha_i^1 - p_i/t = 0, \tag{3}$$

and hence, its best response function

$$p_i(p_j, \alpha_i^e; \alpha_i^0) = \frac{t\alpha_i^0 + b(2\alpha_i^e - 1) + p_j}{2} \text{ for } i = A, B \text{ and } i \neq j. \tag{4}$$

Solving firms' best response functions and substituting $\alpha_j = 1 - \alpha_i$ $(j \neq i)$, yields firms' profit maximizing prices

$$p_i(\alpha_i^e; \alpha_i^0) = \frac{t(\alpha_i^0 + 1) + b(2\alpha_i^e - 1)}{3} \text{ for } i = A, B \text{ and } i \neq j. \tag{5}$$

Substituting (5) for $i, j = A, B$ into Condition (3) and substituting $k := t/b$, gives the reduced demand functions

$$\alpha_i^1(\alpha_i^e; \alpha_i^0) = \frac{k(\alpha_i^0 + 1) + 2\alpha_i^e - 1}{3k} \text{ for } i = A, B \text{ and } i \neq j. \tag{6}$$

In a fulfilled expectations equilibrium it must hold that consumers expectations about market shares are fulfilled; i.e., we require that $\alpha_i^I(\alpha_i^e; \alpha_i^0) = \alpha_i^e$ (the index "I" stands for the *interior* market sharing equilibrium) holds for $i = A, B$. Applying this condition to Equation (6) yields the equilibrium market share of firm i in the market sharing outcome

$$\alpha_i^I(\alpha_i^0, k) = \frac{k(1 + \alpha_i^0) - 1}{3k - 2} \text{ for } i = A, B. \tag{7}$$

From inspecting Equation (7) we observe that an interior solution does not exist if $k = 2/3$. Equation (7) also shows that firms' equilibrium market shares only depend on their initial market shares and the parameter k. Existence of the market sharing solution is guaranteed if and only if

$$0 < \alpha_i^I(\alpha_i^0, k) < 1 \tag{8}$$

holds. We are now in a position to prove the following lemma.[19]

Lemma 1. *A unique market sharing equilibrium exists, where firms' market shares and prices are given by* $\alpha_i^I = \left[k(1 + \alpha_i^0) - 1\right]/(3k - 2)$ *and* $p_i^I = t\alpha_i^I$*, respectively, if and only if either*

[19] In the following we rule out the case $k = 2/3$, where an interior solution does not exist. Of course, in the following we also consider only the relevant parameter space with $k > 0$ and $\alpha_i^0 \in [0, 1]$, for $i = A, B$.

$\alpha_i^0 \in (\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$ or $\alpha_i^0 \in (\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k))$ holds ($i = A, B$), with $\overline{\alpha}^0(k) := 2 - 1/k$ and $\overline{\overline{\alpha}}^0(k) := 1/k - 1$. Moreover, $\partial\overline{\alpha}^0/\partial k > 0$, $\partial\overline{\overline{\alpha}}^0/\partial k < 0$, $\lim_{k\to(2/3)} \overline{\alpha}^0 = \lim_{k\to(2/3)} \overline{\overline{\alpha}}^0 = 1/2$, $\overline{\alpha}^0(1) = \overline{\overline{\alpha}}^0(1/2) = 1$, and $\overline{\alpha}^0(1/2) = \overline{\overline{\alpha}}^0(1) = 0$.

Proof. First notice that market shares add up to unit; hence, if (8) holds, then $0 < \alpha_j^I(\alpha_j^0, k) < 1$ holds as well, with $i, j = A, B$ and $i \neq j$. Hence, existence of the interior solution (7) is guaranteed if and only if Condition (8) holds. Note also that Condition (8) implies $p_i^I > 0$ ($i = A, B$). We first prove that for $k < 2/3$ the market sharing equilibrium arises if $\alpha_i^0 \in (\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$. We then prove that for all $k > 2/3$ a market sharing equilibrium exists if $\alpha_i^0 \in (\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k))$.

Case i) ($k < 2/3$). Applying Condition (8) gives that $\alpha_i^I > 0 \Leftrightarrow \alpha_i^0 < 1/k - 1$ while $\alpha_i^I < 1 \Leftrightarrow \alpha_i^0 > 2 - 1/k$.

Case ii) ($k > 2/3$). Again, using Condition (8) gives that $\alpha_i^I > 0 \Leftrightarrow \alpha_i^0 > 1/k - 1$ and $\alpha_i^I < 1 \Leftrightarrow \alpha_i^0 < 2 - 1/k$.

Differentiation of the threshold values $\overline{\alpha}^0(k)$ and $\overline{\overline{\alpha}}^0(k)$ gives $1/k^2 > 0$ and $-1/k^2 < 0$, respectively. Finally, uniqueness follows from the concavity of firms' optimization problems over the relevant parameter range. *Q.E.D.*

Lemma 1 shows that a unique market sharing equilibrium exists for a large parameter range. For example, if $k < 1/2$ or $k > 1$, then a market sharing equilibrium always exists independently of the distribution of firms' installed bases. This is not necessarily the case for intermediate values of k. Precisely, for $k \in [1/2, 1]$ a market sharing outcome does not exist, if the distribution of firms' installed bases is sufficiently asymmetric such that either $\alpha_A^0 \geq \max\{\overline{\alpha}^0, \overline{\overline{\alpha}}^0\}$ or $\alpha_A^0 \leq \min\{\overline{\alpha}^0, \overline{\overline{\alpha}}^0\}$ holds. We now turn to the analysis of the monopoly equilibrium.

Monopoly outcome. In a monopoly equilibrium where one firm gains the entire market (say firm A), it must hold that $\alpha_A^e = \alpha_A^M = 1$ (the index "M" stands for the monopoly equilibrium). Clearly, the price of firm A, p_A, then follows from setting $U_{x=1}^A = U_{x=1}^B$, such that the marginal consumer is located at the other end of the unit interval; i.e., at the point $x = 1$. Otherwise, if $U_1^A > U_1^B$, then firm A could increase its profit by increasing its price and if $U_1^A < U_1^B$, then firm A would not gain the entire market, with $\alpha_A^M = 1$. The rival firm B can not do better than setting $p_B = 0$, because for positive prices $p_B > 0$ firm B may increase its profit by lowering its price. Equating U_x^A and U_x^B either at $x = 0$ or $x = 1$ yields the price of firm i ($i = A, B$) in the monopoly equilibrium

$$p_i^M(\alpha_i^0) = b - t(1 - \alpha_i^0), \tag{9}$$

when firm i becomes the monopolist and firm j ($j \neq i$) is driven off the market. The price $p_i^M(\alpha_i^0)$ (together with $p_j^M = 0$, with $j \neq i$) can only constitute an equilibrium if it is nonnegative, so that

$$k(1 - \alpha_i^0) \leq 1 \tag{10}$$

must hold. Moreover, firm i must not have an incentive to increase its price above the price given by (9). By increasing the price firm i faces the demand as given by (1) and its profit is then given by $\pi_i(p_i) = p_i(\alpha_i^0 t - p_i + b)/t$ as $p_j = 0$ and $\alpha_i^e = 1$ must hold in the monopoly equilibrium. We guarantee that firm i does not have an incentive to increase its price if

$$\left.\frac{\partial \pi_i}{\partial p_i}\right|_{p_i = b - t(1-\alpha_i^0)} = 2 - \alpha_i^0 - 1/k \leq 0$$

holds. Rewriting this condition gives

$$k(2 - \alpha_i^0) \leq 1 \text{ for } i = A, B. \tag{11}$$

Obviously, Condition (11) is binding when compared with Condition (10). Substituting the installed bases, α_i^0 and α_j^0 ($i, j = A, B$ and $i \neq j$), into (11) we obtain that a monopoly equilibrium exists with firm i (firm j) gaining the whole market, if $\alpha_i^0 \geq \overline{\alpha}^0$ ($\alpha_i^0 \leq \overline{\overline{\alpha}}^0$) holds. We summarize our results in the following lemma.

Lemma 2. *A monopoly equilibrium with $\alpha_i^M = 1$ ($\alpha_j^M = 1$) exists ($i, j = A, B$ and $i \neq j$), if $\alpha_i^0 \geq \overline{\alpha}^0(k)$ ($\alpha_i^0 \leq \overline{\overline{\alpha}}^0(k)$). The monopoly price of the winning firm is given by $p_i^M(\alpha_i^0) = b - t(1 - \alpha_i^0)$, while the losing firm cannot do better than setting $p_j = 0$. In that area the following constellations emerge:*

i) Multiple monopoly equilibria: If $\alpha_i^0 \in [\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)]$, then both $\alpha_i^M = 1$ and $\alpha_j^M = 1$ ($i \neq j$) are equilibrium outcomes.

ii) Unique monopoly equilibrium: If $\alpha_i^0 > \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ or if $\alpha_i^0 = \overline{\alpha}^0(k)$ for all $k \in (2/3, 1]$, then $\alpha_i^M = 1$ is the unique monopoly equilibrium. If $\alpha_i^0 < \min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ or if $\alpha_i^0 = \overline{\overline{\alpha}}^0(k)$ for all $k \in (2/3, 1]$, then $\alpha_j^M = 1$ ($i \neq j$) is the unique monopoly equilibrium.

Combining Lemma 1 and 2, we can fully characterize the equilibrium pattern in the next proposition.

Proposition 1. *The following equilibrium constellations emerge.*

i) Monopoly and market sharing equilibria. If $\alpha_i^0 \in (\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$, *then* $\alpha_i^M = 1$, $\alpha_j^M = 1$ *and* $\alpha_i^I = \alpha_i^I(k, \alpha_i^0)$ *for* $i = A, B$ *and* $i \neq j$ *are equilibria.*

ii) Unique market sharing equilibrium: If $\alpha_i^0 \in (\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k))$, *then* $\alpha_i^I = \alpha_i^I(\alpha_i^0, k)$ *for* $i = A, B$ *is the unique equilibrium.*

iii) Unique monopoly equilibrium: If $\alpha_i^0 > \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ *or if* $\alpha_i^0 = \overline{\alpha}^0(k)$ *for all* $k \in (2/3, 1]$, *then* $\alpha_i^M = 1$ *(*$i = A, B$*) is the unique monopoly equilibrium. If* $\alpha_i^0 < \min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ *or if* $\alpha_i^0 = \overline{\overline{\alpha}}^0(k)$ *for all* $k \in (2/3, 1]$, *then* $\alpha_j^M = 1$ *(*$i, j = A, B$ *and* $i \neq j$*) is the unique monopoly equilibrium.*

iv) Multiple monopoly equilibria: Both $\alpha_i^M = 1$ *and* $\alpha_j^M = 1$ *(*$i, j = A, B$ *and* $i \neq j$*) are the only equilibria, if* $\alpha_i^0 \in \{\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k)\}$ *for all* $k \in [1/2, 2/3)$.

It is instructive to interpret Proposition 1 in terms of the switching cost-network effect ratio, k. As k increases proportionally with switching costs, we can distinguish three cases: *i)* "high switching costs" for $k > 1$, *ii)* "moderate switching costs" for $1/2 < k \leq 1$, and *iii)* "low switching costs" for $k \leq 1/2$.[20] Proposition 1 shows that the equilibrium pattern (monopolization *vs.* market sharing and unique equilibrium *vs.* multiple equilibria) depends on the parameter k and firms' installed bases.

Let us first consider the region, where switching costs are low ($k \leq 1/2$). In that area, network effects dominate which gives rise to multiple equilibria. Depending on consumer expectations both a monopoly outcome and a market sharing outcome are possible. In that sense, we obtain qualitatively the same pattern as in Katz and Shapiro (1985), where the coexistence of symmetric and asymmetric equilibrium outcomes has been shown for the case of Cournot competition between incompatible technologies. Strong network effects allow for the possibility of a monopoly outcome whenever consumers expect monopolization. The rival firm cannot break that equilibrium even though switching costs are low. If, however, consumers expect a market sharing outcome, then both firms obtain positive market shares. Interestingly, this result does not depend on the size of firms' installed bases. A large installed base does not "tip" the market necessarily into the monopoly outcome; again, if consumers do not expect a firm to monopolize

[20] To simplify we do not discuss the somehow special cases where $\alpha_i^0 \in \{\overline{\alpha}^0, \overline{\overline{\alpha}}^0\}$ for $1/2 \leq k < 2/3$, where only the two monopoly equilibria emerge.

the market.

We now turn to the case of high switching costs ($k > 1$), where market sharing constitutes the unique equilibrium outcome. This result shows that the *relative* importance of network effects and switching costs is critical to understand market dynamics. Neglecting, for instance, switching costs and focusing instead exclusively on network effects in order to predict the likely dynamics of a certain "network" industry, may lead to false conclusions. A preoccupation with network effects leads one to conclude that the market behaves "tippy" (see Shapiro and Varian, 1998) and is likely to be monopolized by one of the technologies, while actually the market remains in a stable market sharing equilibrium because of high switching costs. Similar to Beggs and Klemperer (1992), in markets with high switching costs competition is attenuated such that a dominant firm prefers to exploit its installed base and therefore, allows the rival firm to gain market share. From a consumer perspective, a monopolizing outcome (where network effects are maximized) becomes less attractive as switching costs increase. In fact, from Lemma 1, we know that firm i's price is given by $p_i^I = t\alpha_i^I$, so that consumers' total expenses are given by $t\left[(\alpha_A^I)^2 + (1-\alpha_A^I)^2\right]$ which obtains a maximum when one of the firms serves the entire market. Conversely, overall expenses are minimized, whenever firms share the market equally. Therefore, if switching costs are high, then consumers benefit from a market sharing outcome.

Taking our results for small and high switching together, we observe that our model nests two important views on markets with network effects and switching costs: First, if network effects dominate ($k \leq 1/2$), then similar results as derived in Katz and Shapiro (1985) emerge, while for cases where switching costs dominate ($k > 1$) results from the switching costs literature (Beggs and Klemperer, 1992) remain valid.

Let us now examine the intermediate range, with $1/2 < k \leq 1$, where switching costs are moderate, so that both network effects and switching are more balanced. In that region we obtain strikingly different market dynamics, neither captured in the network effects nor in the switching costs literature. First of all, in that area the installed base plays a critical role in determining the further development of the market. Most importantly, Proposition 1 allows us to derive an important result on the contentious issue of consumer lock-in which is also closely related to the so-called "critical mass" effect in network industries. A critical mass effect occurs when a firm's market share becomes so large that consumers become inevitably trapped in that technology for ever. As much of the literature on technology adoption in markets with network effects assumes perfect lock-in of consumers (see, e.g., Farrell and Saloner, 1986, or Arthur, 1989), one may argue that a critical mass effect only occurs for very large switching costs. In contrast, our analysis of the interplay of network effects and switching costs under Bertrand competition reveals that rather small (but not too small) switching costs are more likely to create a critical mass effect than large switching cost. The following corollary states our result concerning the existence of a critical mass, $\widetilde{\alpha}_i^0$, for firm i, such that the unique equilibrium outcome is the monopoly outcome with $\alpha_i^M = 1$.

Corollary 1. *One firm holds a critical mass of consumers, $\widetilde{\alpha}_i^0$, and therefore, becomes the monopolist, with $\alpha_i^M = 1$, for sure (as a unique equilibrium outcome) either if $\widetilde{\alpha}_i^0 > \overline{\overline{\alpha}}^0(k)$ or $\widetilde{\alpha}_i^0 < \overline{\alpha}^0(k)$ for all $k \in [1/2, 2/3)$, or if $\widetilde{\alpha}_i^0 \geq \overline{\alpha}^0(k)$ or $\widetilde{\alpha}_i^0 \leq \overline{\overline{\alpha}}^0(k)$ for all $k \in (2/3, 1]$. The critical mass always fulfills $\widetilde{\alpha}_i^0 > 1/2$, with $i = A, B$.*

Corollary 1 shows that a lock-in into one of the technologies may only occur for "intermediate" values of the switching cost-network effect ratio. Put another way, for large switching costs which involve parameter values $k > 1$, a critical mass effect cannot occur for sure. In that range we obtain that the market sharing outcome is the unique equilibrium outcome. Assuming perfect lock-in as a proxy for switching costs can, therefore, lead to false conclusions. If markets are imperfectly competitive, then large switching costs evoke a fat cat effect (similar to the "bargain-then-ripoffs" mechanism in the switching costs literature) that works in favor of a market sharing outcome. From a consumer perspective the monopoly outcome is attractive for moderate switching costs, as this maximizes network effects and switching costs are not too large in that region. Interestingly, in order to lock-in consumers for sure, large network effects ($k \leq 1/2$) alone cannot make it. If network effects are large and the costs of switching are negligible, then consumers can always afford to switch to the other firm. As a consequence, a perfect lock-in of consumers is never admissible in that case. Our analysis, therefore, shows that network effects are an important driver that leads to the monopolization of markets. However, consumer lock-in can only occur in the presence of switching costs such that both market forces remain balanced. When network effects are large and switching costs are low, then multiple equilibria emerge, so that consumers never become necessarily locked-in for ever. In those instances, changes in consumer expectations may lead to erratic changes such that the rival firm may monopolize the market or market sharing occurs in the future.

If none of the firms has reached the critical mass, then the type of the equilibrium for moderate switching costs depends on the exact value of k. If switching costs are rather low (i.e., $1/2 < k < 2/3$ holds), then the equilibrium pattern is similar to the case of small switching costs ($k \leq 1/2$). For larger switching costs (with $2/3 < k \leq 1$) the equilibrium is similar to the case of high switching costs, such that market sharing prevails. Interestingly, if we approach the point $k = 2/3$ (either from below or above) the monopolization outcome becomes more and more likely, such that in the limit the area completely vanishes, where market sharing is possible.

Let us now have a closer look at the market dynamics in the market sharing equilibrium. We are interested whether the initially dominant firm always keeps its dominant position or whether the initially smaller rival firm can also become dominant. We are also interested in the asymmetry of the market outcomes; namely, is the total value of the difference of firms' market shares increasing or decreasing over time? With respect to the first property we distinguish between "monotone" and "alternating" market dynamics, where the former (latter) case refers to a situation where the dominant firm keeps (loses) its dominant position. With respect to the second property we distinguish "monopolization" and "market sharing," where the former (latter) case means that the difference of market shares widens (narrows).

Proposition 2. *Consider the parameter range where market sharing is an equilibrium outcome and assume $\alpha_i^0 \neq 1/2$. We can then distinguish four different market dynamics:*

i) Monotone market sharing. If $k > 1$, then the initially dominant firm, i, loses market share but keeps its dominant position; i.e., $\alpha_i^0 > \alpha_i^I > 1/2 > \alpha_j^I > \alpha_j^0$, for $i, j = A, B$ and $i \neq j$.

ii) Monotone monopolization. If $k \in (2/3, 1)$, then the market share of the initially dominant firm, i, increases; i.e., $\alpha_i^I > \alpha_i^0 > 1/2 > \alpha_j^0 > \alpha_j^I$, for $i, j = A, B$ and $i \neq j$.

iii) Alternating monopolization. If $k \in (1/2, 2/3)$, then the initially dominant firm, i, loses its dominant position and the share of the rival firm, j, is larger than the initial share of the dominant firm; i.e., $\alpha_j^I > \alpha_i^0 > 1/2 > \alpha_j^0 > \alpha_i^I$, for $i, j = A, B$ and $i \neq j$.

iv) Alternating market sharing. If $0 < k < 1/2$, then the initially dominant firm, i, loses its dominant position and the share of the rival firm, j, is smaller than the initial share of the dominant firm; i.e., $\alpha_i^0 > \alpha_j^I > 1/2 > \alpha_i^I > \alpha_j^0$, for $i, j = A, B$ and $i \neq j$.

Moreover, if $\alpha_i^0 = 1/2$, then $\alpha_i^I = 1/2$, with $i = A, B$. If $k = 1/2$ and $\alpha_i^0 > 0$, then

$\alpha_i^I = 1 - \alpha_i^0$ *for* $i = A, B$. *If* $k = 1$ *and* $\alpha_i^0 > 0$, *then* $\alpha_i^I = \alpha_i^0$ *for* $i = A, B$.

Proof. We have to compare firms' initial market shares with the realized market shares in the market sharing equilibrium which are given by Equation (7) (note that the relevant parameter space is specified in Lemma 1). Suppose that $\alpha_i^0 \neq 1/2$. A firm (say, firm i) obtains a dominant position if $\alpha_i^I = (k-1+k\alpha_i^0)/(3k-2) > 1/2$ holds. This can only be the case, if either $\alpha_i^0 > 1/2$ and $k > 2/3$ or $\alpha_i^0 < 1/2$ and $k < 2/3$ hold. Hence, for all $k > 2/3$ the initially dominant firm keeps its dominant position while for $k < 2/3$ the initially dominant firm loses its dominant position to the rival firm. We now examine whether market shares converge to a market sharing outcome (with $|\alpha_i^0 - \alpha_j^0| > |\alpha_i^I - \alpha_j^I|$) or whether the total value of the difference of firms' market shares increases (with $|\alpha_i^0 - \alpha_j^0| < |\alpha_i^I - \alpha_j^I|$) such that there is a trend towards monopolization (with $i, j = A, B$ and $i \neq j$). We obtain that $|\alpha_i^0 - \alpha_j^0| > |\alpha_i^I - \alpha_j^I|$ holds if and only if $k < 1/2$ or $k > 1$, while $|\alpha_i^0 - \alpha_j^0| < |\alpha_i^I - \alpha_j^I|$ is true if and only if $1/2 < k < 1$ (note that $k \neq 2/3$). Combining those results, we obtain all four patterns as specified in the proposition.

The last part of the proposition follows directly from substituting the specific values into Equation (7). *Q.E.D.*

Proposition 2 shows that changes in the market shares of the firms in the market sharing outcome are determined by the ratio of switching costs to network effects, k. If switching costs are high ($k > 1$), then the dominant firm loses market shares but remains dominant. This result is similar to Beggs and Klemperer (1992), where it has been shown that a market with consumer switching costs should converge monotonically towards a stable market sharing outcome after a shock. For high switching costs consumers are not very eager to switch and network effects play only a minor role. As a result the market converges to a market sharing outcome in a monotone way.

Turning to the case of small switching costs ($k < 1/2$) Proposition 2 states that convergence towards a market sharing outcome can also be alternating. In that area the firm with the smaller installed base becomes dominant, but its new market share is smaller than the initial market share of the formerly dominant firm. Consumers cannot expect a larger market share for the new dominant firm, as in that case the monopoly outcome is inevitable because of low switching costs. Moreover, an outcome similar to the one with high switching costs is not rational in that area, because this would take away competitive pressure from the firms. As switching costs are small, keeping the competitive pressure between firms is optimal for consumers, which in turn, induces significant consumer switching in equilibrium. As a result, market dominance alters, while the market dynamics tend towards market sharing.

For moderate switching costs ($1/2 < k < 1$) the dynamics depend on the exact value of k. If switching costs are rather high in that region (i.e., $2/3 < k < 1$), then the market sharing outcome converges monotonically towards monopolization as the difference between firms' market shares becomes larger at the end of the period. In that area, consumer are not too eager to switch and find it optimal to enjoy higher network at relatively high prices charged by the dominant firm rather than lower network effects and a lower price from the smaller rival firm. If switching costs are moderate and rather small in that region (i.e., $1/2 < k < 2/3$), then dominance alternates, but in contrast to the case of small switching costs, the market share of the new dominant firm is strictly larger than the initial market share of the formerly dominant firm. In that area, consumers can expect a larger market share in the market sharing equilibrium when compared with the case of small switching costs ($k < 1/2$) because of higher switching costs. Proposition 2 also shows for a particular case (precisely, $k = 1/2$ and $\alpha_i^0 > 0$) that the firms may interchange market shares in each period, a pattern similar to the alternating dominance outcome in Farrell and Shapiro (1998). Moreover, if firms are symmetric ex ante (i.e., $\alpha_i^0 = 1/2$), then the market remains in the equal market sharing equilibrium. This result is also suggested in Katz and Shapiro (1985), where firms are assumed to be symmetric, and hence, obtain equal market shares in the symmetric (interior) equilibrium. Our analysis, however, shows that even small asymmetries among firms (in terms of their installed bases) may have drastic consequences. While for small and high switching costs (i.e., $k < 1/2$ and $k > 1$) the market tends towards equal market sharing, we obtain that asymmetries tend to increase for moderate switching costs ($1/2 < k < 1$).

Before we elaborate further on market dynamics in the next section, we now examine the social welfare and consumer surplus consequences of our basic model. Firstly, we compare consumer surplus and social welfare under the monopoly equilibria and the market sharing equilibrium when both equilibria coexist. Secondly, we ask how consumer surplus and social welfare change in the market sharing equilibrium. As we will see, in both instances a fundamental conflict between social welfare and consumer surplus maximization prevails.

The next proposition summarizes our results for the comparison of social welfare and consumer surplus under the market sharing equilibrium and both monopoly equilibria.

Proposition 3. *Consider the parameter region where both the market sharing equilibrium and the monopoly equilibria coexist. Then, social welfare is always higher in the monopoly equilibria when compared with the market sharing equilibrium. For the comparison of consumer surplus we obtain the following cases:*

i) If $k \leq 1/2$, then consumer surplus is always higher in the market sharing equilibrium when compared with both monopoly equilibria.

ii) Let $k \in (1/2, 2/3)$ and suppose that either one of the firms becomes the monopolist in the monopoly equilibrium. Then there exists a unique threshold value $\widehat{\alpha}^0(k)$ $(1 - \widehat{\alpha}^0(k))$, with $\widehat{\alpha}^0(k) := [k(13 - 10k) - 4]/k^2$, such that consumer surplus is higher in the market sharing equilibrium when compared with the monopoly equilibrium where $\alpha_i^M = 1$ $(\alpha_j^M = 1)$, if $\alpha_i^0 > \widehat{\alpha}^0(k)$ $(\alpha_i^0 < 1 - \widehat{\alpha}^0(k))$ with $i, j = A, B$ and $i \neq j$. The opposite holds if $\alpha_i^0 < \widehat{\alpha}^0(k)$ $(\alpha_i^0 > 1 - \widehat{\alpha}^0(k))$, while indifference holds for $\alpha_i^0 = \widehat{\alpha}^0(k)$ $(\alpha_i^0 = 1 - \widehat{\alpha}^0(k))$. Moreover, $\widehat{\alpha}^0(k)$ $(1 - \widehat{\alpha}^0(k))$ is strictly concave (convex) over $k \in (1/2, 2/3)$, reaches its maximum (minimum) at $k = 8/13$ with $\widehat{\alpha}^0(8/13) = 9/16$ $(1 - \widehat{\alpha}^0(8/13) = 7/16)$, while $\widehat{\alpha}^0(1/2) = 0$ and $\lim_{k \to 2/3} \widehat{\alpha}^0(k) = 1/2$ hold.

iii) Let $k \in (1/2, 2/3)$ and suppose that both firms $i = A, B$ may become the monopolist in the monopoly equilibrium. Then, for all $\alpha_i^0 \in (\widehat{\alpha}^0, 1 - \widehat{\alpha}^0)$ which implies $k \in (1/2, 4/7]$, consumer surplus is higher in the market sharing equilibrium when compared with both monopoly equilibria, while in all other instances either one of the monopoly equilibria gives rise to a higher consumer surplus when compared with market sharing equilibrium. Moreover, if $\alpha_i^0 \in (\widehat{\alpha}^0(k), 1 - \widehat{\alpha}^0(k))$ which implies $k \in (4/7, 2/3)$, then both monopoly equilibria give rise to a strictly higher consumer surplus than the market sharing equilibrium.

Proof. See Appendix.

Proposition 3 states that social welfare is always lower in the market sharing equilibrium when compared with the monopoly outcome. The main reason for this result is that network effects are maximized in the monopoly outcome. As switching costs are relatively small in the area where both types of equilibria coexist, we obtain that a monopoly outcome is always preferred from a social welfare point of view. Most importantly, Proposition 3 reveals a fundamental conflict between social surplus and consumer surplus. The conflict becomes most obvious when we consider the parameter region where network effects dominate switching costs (i.e., $k \leq 1/2$ holds). In that region consumers strictly prefer market sharing to a monopoly outcome as market sharing minimizes consumers' overall payments to the firms. Interestingly, the result is independent of firms' installed bases, so that even significant consumer switching in the market sharing equilibrium does not affect the ordering.

The tension between social welfare and consumer surplus remains to some extent valid in the parameter range, where switching costs become larger (i.e., $1/2 < k < 2/3$). Precisely, we obtain a critical value for firm i's initial market share, $\widehat{\alpha}^0(k)$, such that consumer welfare is maximized under the monopoly outcome (with firm i monopolizing the market) if firm i's initial market share does not fall short of the critical value. Hence, consumers can be better off in the monopoly equilibrium when compared with the market sharing equilibrium if the prospective monopolist has a relatively small installed base and must, therefore, price aggressively (i.e., set a relatively low price) in order to obtain the (expected) monopoly position. If, however, the initial market share of the prospective monopolist is larger than the critical market share, $\widehat{\alpha}^0(k)$, we obtain that consumer surplus is largest in the market sharing equilibrium. Again, the reason for this result is that if the prospective monopolist's initial market share is already large, then its pricing behavior is less aggressive in order to sustain the monopolization of the market. Consequently, the monopoly outcome is less attractive for the consumers in those instances, so that market sharing maximizes consumers' overall welfare.

The third part of Proposition 3 compares consumer surplus under the market sharing equilibrium with both monopoly equilibria. The region where the market sharing outcome maximizes consumer surplus when compared with both possible monopoly outcomes vanishes if switching costs become sufficiently large (i.e., at the point $k = 4/7$). As switching cost become larger sustaining the market sharing equilibrium becomes increasingly costly as market sharing involves substantial switching costs (recall Proposition 2 where we have shown alternating dynamics in the market sharing equilibrium in that region). Interestingly, in the interval $k \in (4/7, 2/3)$ there also exists an area for installed bases close to market sharing such that both monopoly outcomes give rise to higher consumer surpluses than the market sharing equilibrium, so that social welfare and consumer surplus maximization are aligned in that area.

Our results are instructive for recent policy debates that circle around the appropriate application of traditional competition policy instruments in markets with pronounced network effects (see, e.g., OECD, 1997, and FTC, 1996). While some consensus has been reached concerning the desirability of compatibility, the assessment of market outcomes when products are incompatible remains largely unresolved (see also Klemperer, 2005). Incompatibilities give rise to ambiguities as on the one hand pronounced network effects may drive the industry towards monopolization (an obviously unfortunate outcome from a traditional competition policy point of view) while on the other hand under a market sharing outcome where incompatible products compete head-to-head substantial incompatibilities among consumers prevail (an outcome being obviously inefficient).

As Proposition 3 shows at least some of the ambiguities concerning the policy assessment of competition under incompatible products can be attributed to a fundamental conflict between consumer welfare and social welfare. If network effects are sufficiently large, consumers prefer the market sharing equilibrium (where price competition is most intense) over the monopoly outcome, while a social planer would prefer either one of the monopoly equilibria (where network effects are maximized).[21] Taking a policy making perspective, our results highlight the trade-off involved with those governmental interventions which aim at picking a winning proprietary technology out of incompatible competitors (e.g., by committing governmental procurement or standard setting to a single technology).[22] While such a policy can be advisable from a social welfare perspective, consumers may be substantially hurt.[23] Our results also show that

[21] Our finding is related to Farrell and Saloner (1992) who showed in a model of technology competition under network effects that the existence of (imperfect) converters makes a standardization (or, equivalently, a monopoly) outcome less likely, so that overall incompatibilities tend to be larger with converters. They interpret their finding as an inefficiency due to the "irresponsibility of competition"; a phenomenon which occurs quite generally under (incompatible) duopoly competition (see Suleymanova and Wey, 2008).

[22] A recent example for this kind of intervention can be seen in the announcement of the EU to support DVB-H as the mobile-television standard over rival technologies, as e.g., Qualcomm's MediaFLO ("EU Opts for DVB-H as Mobile-TV Standard," The Wall Street Journal Europe, March 18, 2008, p. 5).

[23] One may speculate that our results are somehow supported by the fact that policy makers taking an industrial policy perspective (i.e., focus primarily on profits) tend to prefer to pick a winning technology (out of a set of incompatible alternatives) while in competition policy circles (which are supposed to focus primarily on consumer surplus) a more reticent attitude appears to have gained control (as, e.g., in FTC, 1996).

the fundamental conflict between consumer surplus and social welfare tends to vanish when switching costs become relatively more important. Therefore, in industries where both network effects and switching costs are important, a monopoly outcome can be preferable both from a social welfare and a consumer surplus perspective (which may have been the case in the above mentioned DVD format war, where Toshiba decided to pull out recently).

We now turn to the question how consumer surplus and social welfare change in the market sharing equilibrium. Our results are summarized in the following proposition.

Proposition 4. *If $\alpha_i^0 \neq 1/2$ and $k \notin \{1/2, 1\}$, then consumer surplus strictly increases in the market sharing equilibrium, while it does not change if $\alpha_i^0 = 1/2$ or if $k \in \{1/2, 1\}$. Social welfare in the market sharing equilibrium decreases for all $k \in \left(\left(3-\sqrt{5}\right)/2, 1/2\right)$ and $k \in \left(1, \left(3+\sqrt{5}\right)/2\right)$, does not change if $\alpha_i^0 = 1/2$ or if $k \in \left\{\left(3-\sqrt{5}\right)/2, 1/2, 1, \left(3+\sqrt{5}\right)/2\right\}$, and increases otherwise.*

Proof. See Appendix.

As a rule, Proposition 4 shows that consumer surplus increases in the market sharing equilibrium when compared with the consumer surplus that would have prevailed if the initial distribution of firms' market shares had been an equilibrium outcome. Comparing the development of social welfare in the same way, Proposition 4 shows that overall welfare may decrease or increase depending on the parameter k. If switching costs are sufficiently large (i.e., $k > \left(3+\sqrt{5}\right)/2$), then switching costs tend to decrease, so that social welfare increases besides a reduction in network effects, where the latter follows from the fact the market dynamics converge towards equal market sharing in that area. If switching cost give rise to a lower k, with $k \in \left(1, \left(3+\sqrt{5}\right)/2\right)$, then social welfare tends to decrease as the convergence towards market sharing reduces network effects which are relatively more important when compared with the previous case. Interestingly, the dynamics of consumer surplus and social welfare are aligned for intermediate switching costs (i.e., $1/2 < k < 1$) because network effects tend to increase in that area (recall that market dynamics always tend towards monopolization if $1/2 < k < 1$). For small switching costs (i.e., $k < 1/2$) market dynamics are alternating and converge towards market sharing. Consequently, social welfare can only increase in that area if switching costs are relatively small (i.e., $k < 3-\sqrt{5}/2$), while in the remaining instances social welfare tends to decrease.

We are aware that our results concerning the dynamics of consumer and social surplus in the market sharing equilibrium have to be interpreted cautiously as we did not specify a fully dynamic model. Nevertheless, Proposition 4 indicates that the comparison of market sharing outcomes from period to period may give rise to conflicting evaluations depending on whether a consumer or a social welfare perspective is taken.

4 Extensions

In the following we consider three extensions of our basic model. *First*, we analyze stationary and stable equilibria if our market game is infinitely often repeated. *Second*, we analyze firms' incentives to make their products compatible. And *third*, we analyze firms' incentives to increase switching costs.

4.1 Market Dynamics with Myopic Behavior

In the previous analysis we have analyzed how firms market shares change within one period. In this section we analyze the dynamic extension of our one-period game. We assume that the market game is infinitely often repeated. We denote the initial market share of firm i ($i = A, B$) in each period $n \in [1, \infty)$ by α_i^{n-1} and at the end of the period by α_i^n. The initial market share at period $n = 1$ is then α_i^0. We assume that both the consumers and the firms are myopic;[24] this is, firms maximize their per period profits and consumers their per period utilities. We are interested whether the sequence of market games converges towards a stable and stationary equilibrium outcome, and how such an equilibrium looks like. A stationary equilibrium is reached if $\alpha_i^{n-1} = \alpha_i^n$ ($i = A, B$), is the unique equilibrium outcome, so that market shares do not change anymore for sure. In addition, a stable equilibrium is robust to small changes of firms' market shares. Accordingly, an equilibrium is stable if there exists a neighborhood of the (stationary) equilibrium market share such that if the initial market share lies within this neighborhood then at the end of the period the equilibrium market share will be closer to the stationary equilibrium than the initial market share. Using our previous results, we obtain the following proposition which characterizes the stable and stationary equilibria of the infinitely repeated market game.

[24]Myopic behavior can be a good approximation for rational behavior, if agents have high discounting factors and/or the product life cycle is quite long and the industry is regularly disturbed by shocks which make intertemporal optimization useless.

Proposition 5. *There exists no stationary and stable equilibrium if switching cost are small (i.e., $k \leq 1/2$) or if $k = 1$. For all other parameter values of k there exists a unique stationary and stable equilibrium with the following properties:*

i) For high switching costs ($k > 1$) firms share the market equally.

ii) For moderate switching costs ($1/2 < k < 1$) one firm serves the entire market.

Proof. We prove first the non-existence of a stationary and stable equilibrium for low switching costs ($k < 1/2$). According to Proposition 1 for $k < 1/2$ and any $\alpha_i^0 \in [0,1]$ multiple equilibria prevail. Hence, there exists no stationary and stable equilibrium. If $k = 1/2$ ($k = 1$) then any α_i^{n-1} would give rise to a new stationary equilibrium with $\alpha_i^n = 1 - \alpha_i^{n-1}$ ($\alpha_i^n = \alpha_i^{n-1}$) for $i = A, B$, so that none of those equilibria can be stable. Let us now consider the remaining cases.

Case i) ($k > 1$). By Proposition 1, for all $k > 1$ and any α_i^0 only the market sharing equilibrium arises with $\alpha_i^I(\alpha_i^0, k)$ given by Equation (7). Substituting recursively initial market shares we can express firm i's market share at the end of period n as a function of the initial market share in $n = 1$; i.e., $\alpha_i^n(\alpha_i^0, k)$. This gives

$$\alpha_i^n(\alpha_i^0, k) = \frac{k-1}{3k-2}\left[1 + ... + \left(\frac{k}{3k-2}\right)^{n-1}\right] + \left(\frac{k}{3k-2}\right)^n \alpha_i^0. \tag{12}$$

Taking the limit of the first term in brackets of Equation (12) we obtain $\frac{(3k-2)}{2(k-1)}$. Accordingly, we obtain for the second term in brackets $\lim_{n\to\infty}\left(\frac{k}{3k-2}\right)^n = 0$. Substituting both expressions into (12) we get

$$\lim_{n\to\infty}\left\{\frac{k-1}{3k-2}\left[1 + ... + \left(\frac{k}{3k-2}\right)^{n-1}\right] + \left(\frac{k}{3k-2}\right)^n \alpha_i^0\right\} = \frac{(k-1)(3k-2)}{(3k-2)2(k-1)} + 0 = \frac{1}{2}.$$

As $\alpha_i^{n-1} = 1/2$ implies $\alpha_i^n = 1/2$ and $\lim_{n\to\infty}\alpha_i^n = 1/2$ holds for any initial market share α_i^0, it follows that market sharing is the unique stationary and stable equilibrium for $k > 1$.

Case ii) ($1/2 < k < 1$). For moderate switching costs either monopoly or market sharing outcomes arise depending on the value of α_i^0. Consider first the case $\alpha_i^{n-1} = 1/2$, which implies $\alpha_i^n = 1/2$ in the considered region. This equilibrium is stationary but not stable, as no neighborhood $\alpha_i^{n-1} \in (1/2 - \epsilon, 1/2 + \epsilon)$, with $\epsilon > 0$, exists such that $\left|\alpha_i^{n-1} - \alpha_i^n\right| < \epsilon$. To the contrary, from Proposition 2, we know that market dynamics always tend towards monopolization in that region. Assume now $\alpha_i^{n-1} \neq 1/2$. According to Proposition 2 in the market sharing equilibrium the share of the initially dominant firm either increases (if $\overline{\overline{\alpha}}^0(k) < \alpha_i^0 < \overline{\alpha}^0(k)$ holds) or the rival firm becomes dominant with a larger market share than the initially dominant firm (if $\overline{\alpha}^0(k) < \alpha_i^0 < \overline{\overline{\alpha}}^0(k)$ holds). Hence, there must exists a period $n' > 1$, where the initial market share of firm i, $\alpha_i^{n'-1}$, fulfills either $\alpha_i^{n'-1} \geq \max\{\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k)\}$ or $\alpha_i^{n'-1} \leq \min\{\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k)\}$. Then we know from Corollary 1 that at the end of period n' one firm must monopolize the market for sure, with $\alpha_i^{n'} = 0$ or $\alpha_i^{n'} = 1$. By Proposition 1 the outcome is stationary as $\alpha_i^n = 0$ ($\alpha_i^0 = 1$) implies $\alpha_i^{n+1} = 0$ ($\alpha_i^{n+1} = 1$). Moreover the equilibrium is stable as no other equilibria emerge if either $\alpha_i^0 > \max\{\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k)\}$ or $\alpha_i^0 < \min\{\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k)\}$ hold. *Q.E.D.*

Proposition 5 shows that for large switching costs ($k > 1$) market sharing is the unique stationary and stable equilibrium. The initial installed base, α_i^0, does not affect the final outcome, such that firms will always share the market equally in that area. For small switching costs, with $k < 1/2$ (and correspondingly relatively high network effects), the multiplicity of equilibria rules out predictable and stable market outcomes (see also Arthur, 1989). If switching costs are moderate (i.e., $1/2 < k < 1$), the monopoly outcome is the only stationary and stable equilibrium. This result follows from the fact that the market sharing equilibrium (if it exists) always tends towards monopolization (as demonstrated in Proposition 2), so that one of the firms inevitably reaches a critical mass, which in turn, drives the market into a stationary and stable equilibrium, where one firm dominates the entire market.

Let us next consider how (exogenous) changes of the switching costs to network effects ratio, k, over time affect the equilibrium in the longer run. Changes of the parameter k may by the result of increasing switching cost which is a typical situation in markets with pronounced network effects, where switching costs are often negligible in the early stages of market development. As consumers start investing into product-specific complementary assets and achieve learning effects by using the technology, switching costs should tend to grow over time. Our model then predicts that an increase of switching costs over time should make a monopoly outcome highly likely, as increases in switching costs drive the industry inevitably into the parameter region where one of the firms obtains a critical mass that forces the industry into the monopoly outcome.

On the other hand, network effects may increase over time in markets where switching costs are important. For example, this may be the case in so-called two-sided market environments, as e.g., online trading platforms, while switching costs do not change over time. Again, our model then predicts that those markets are likely to be driven into the parameter region where switching costs and network effects become more balanced such that the monopoly outcome becomes inevitable. The following corollary summarizes those considerations.

Corollary 2. *If switching costs (network effects) increase over time, then the industry will be driven into a monopoly outcome, whenever the initial value of the parameter, k, is sufficiently small (large); precisely:*

i) Suppose that initially $k < 1/2$ holds. If switching costs increase over time $t_{n+1} - t_n > \epsilon$, with $\epsilon > 0$ small enough, then the industry inevitably reaches the parameter region where the monopoly outcome constitutes the unique stationary and stable equilibrium.

ii) Suppose that initially $k > 1$ holds. If network effects increase over time $b_{n+1} - b_n > \epsilon$, with $\epsilon > 0$ small enough, then the industry inevitably reaches the parameter region where the monopoly outcome constitutes the unique stationary and stable equilibrium.

Corollary 2 follows immediately from Proposition 5. Interestingly, in the case of typewriters the advance of touch typing has been identified by David (1985) as the main reason why the QWERTY keyboard design became the de facto industry standard. Touch typing is, of course, a keyboard specific skill which creates substantial switching costs. More recently, Toshiba decided to pull out of the HD DVD business so that the rival format Blu-ray sponsored by Sony obtains a monopoly position in that market.[25] The decision was announced by Toshiba just after Time Warner decided to support exclusively Blu-ray. Time Warner's decision can be interpreted as an increase in (expected) switching costs, which made the monopoly outcome inevitable.

Corollary 2 is also instructive for markets where the proprietary network effects of firms' products increase over time. Such an evolution may drive the market from a stable market sharing equilibrium into the parameter region where the monopoly outcome constitutes the only stationary and stable equilibrium. One example at hand for this kind of development can be seen in E-Bay's success. E-Bay uses a reputation system where users evaluate sellers' performances. Such a reputation system creates positive network effects which may have grown over time.

We finally, derive consumer surplus and social welfare in both stationary and stable equilibria.

Corollary 3. *In the stable and stationary equilibrium consumer surplus is given by v for*

[25] See "Toshiba is Set to Cede DVD-format Fight," Wall Street Journal Europe, February 18, 2008, p. 3.

moderate switching costs ($k \in (1/2, 1)$) and by $v + (b-t)/2$ (with $(b-t) < 0$) for high switching costs ($k > 1$), while social welfare is given by $v + b$ and $v + b/2$ respectively.

Taking a long run point of view, Corollary 3 states that the stationary and stable equilibrium where one firm monopolizes the entire market is preferable both from a consumer as well as from a social welfare perspective.

4.2 Compatibility Incentives

In this section we analyze firms' incentives to make their products compatible with each other. We assume that compatibility does not erase switching costs. If both firms decide to make their products compatible, then both products become perfect substitutes with respect to their associated network effects. Because of switching costs, both products remain, however, differentiated for consumers who belong to either one of the installed bases.

We use the superscript "c" to denote the case of compatible products. When products are compatible, the amount of network effects which consumers derive from any of the two products is given by b. Consumers still have to bear switching costs if they switch to the rival product. The utility from buying the product of firm i for a consumer with address x under compatibility is then given by

$$U_x^{i,c} = \begin{cases} v + b - p_i, & \text{if} \quad x \in \alpha_i^0 \\ v + b - p_i - t\left|\alpha_A^0 - x\right|, & \text{if} \quad x \in \alpha_j^0, \end{cases} \tag{13}$$

with $i, j = A, B$ and $j \neq i$. From (13) we obtain the demand functions $\alpha_i^c(p_i, p_j; \alpha_i^0)$ under compatibility

$$\alpha_i^c(p_i, p_j; \alpha_i^0) = \begin{cases} 0 & \text{if} \quad p_j - p_i \leq -t\alpha_i^0 \\ \alpha_i^0 + \frac{p_j - p_i}{t} & \text{if} \quad -t\alpha_i^0 < p_j - p_i < t(1-\alpha_i^0) \\ 1 & \text{if} \quad p_j - p_i \geq t(1-\alpha_i^0), \end{cases} \tag{14}$$

with $i, j = A, B$ and $i \neq j$. The following lemma summarizes the equilibrium outcome of the market game.

Lemma 3. *Suppose products are compatible. Then the market sharing equilibrium is the unique equilibrium, where firms' market shares and prices are given by $\alpha_i^c = (1 + \alpha_i^0)/3$ and $p_i^c = t\alpha_i^c$, respectively. Moreover, monotone market sharing prevails everywhere; i.e., $\alpha_i^0 > \alpha_i^c > 1/2 > \alpha_j^c > \alpha_j^0$, for $i, j = A, B$ and $i \neq j$.*

Proof. See Appendix.

Lemma 3 reveals more consistent competitive pattern under compatible products when compared with our previous analysis of incompatible products. Most importantly, monotone market sharing occurs everywhere so that a monopoly outcome is never possible for the case of compatible products.

We now turn to firms' incentives to make their products compatible in the first place. As in Katz and Shapiro (1985) we distinguish two cases depending on whether or not firms can make side payments. While firms are able to maximize their joint surplus with side payments, firms will only agree on compatibility without side payments whenever compatibility involves a Pareto improvement. The next proposition summarizes our results when transfers are ruled out, so that compatibility can only occur if both firms benefit.

Proposition 6. *Firms never agree on making their products compatible with each other if side payments are ruled out. Conflicting incentives arise in the following way:*

i) If under incompatibility the monopoly equilibrium emerges, then the firm which becomes the monopolist loses and the other firm gains from compatibility.

ii) Assume $\alpha_i^0 \neq 1/2$ $(i = A, B)$ *and suppose that the market sharing outcome emerges under incompatibility. Then, depending on the value of the parameter* k *either the dominant or the smaller rival firm loses under compatibility:*

If $k < \frac{2}{3}$*, then the dominant firm gains and the smaller rival firm loses under compatibility.*

If $k > \frac{2}{3}$*, then the dominant firm loses, while the smaller rival firm gains from compatibility.*

Moreover, if both firms share the market equally (i.e., $\alpha_i^0 = 1/2$*, with* $i = A, B$*), then both firms are indifferent between compatibility and incompatibility.*

Proof. See Appendix.

The first part of Proposition 6 shows that the firm which becomes the monopolist under incompatibility does not have an incentive to make the products compatible, while the losing rival firm, of course, prefers compatible product designs. This result is closely related to Katz and Shapiro's (1985) finding that the possibility of an asymmetric equilibrium outcome under incompatibility (which corresponds to the monopoly outcome in our model) should lead to a blockage of compatibility by the "large" firm. The second part of Proposition 6 refers to the market sharing equilibrium under incompatibility. To understand the result it is instructive to analyze how firms' market shares change under compatibility and incompatibility (note that firms' profits are monotone in their market shares). From Proposition 2 we know that the initially dominant firm loses its dominant position under incompatibility if $k < 2/3$, while under compatibility the dominant firm keeps its dominant position (according to Lemma 3). Hence, $\alpha_i^c(\alpha_i^0) > \alpha_i^I(\alpha_i^0)$ must hold for $\alpha_i^0 > 1/2$, so that the dominant firm gains from compatibility. Obviously, in that region the opposite is true for the initially smaller rival firm which, therefore, has an incentive to block a move towards compatibility.

For $2/3 < k < 1$, we know from Proposition 2 that the dominant firm increases its market share under incompatibility, while (according to Lemma 3) it must decrease under compatibility. Hence, the dominant firm loses from a move towards compatibility, while the opposite must be true for the smaller rival firm.

For $k > 1$ the dominant firm loses market shares but still keeps its dominant position both under compatibility and under incompatibility. A comparison of market shares under compatibility $\alpha_i^c(\alpha_i^0) = (1+\alpha_i^0)/3$ and under incompatibility $\alpha_i^I(\alpha_i^0) = (k-1+k\alpha_i^0)/(3k-2)$ yields that $\alpha_i^c(\alpha_i^0) < \alpha_i^I(\alpha_i^0)$ holds for all $k > 2/3$ and $\alpha_i^0 > 1/2$. Hence, the dominant firm loses a larger fraction of its market share under compatibility, and therefore, opposes compatibility. Applying the same logic to the smaller rival firm we obtain conflicting incentives for compatibility.

It is instructive to compare our results with Katz and Shapiro (1985), where it is shown that firms should have an incentive to make their products compatible, whenever under incompatibility the (symmetric) interior solution is realized. In their Cournot model, compatibility leads to an overall expansion of firms' outputs (and hence, an increase in profits) which is absent in our model. It is an artifact of our model that such a market expansion cannot occur in our analysis. However, our analysis of asymmetric installed bases reveals that a fundamental conflict of interests between an initially dominant firm and its smaller rival remains valid in the (interior) market sharing outcome. Overall, our results, therefore, increase the bar for possible market expansion effects so as to make compatibility profitable for both firms when switching costs are present and side payments are not feasible.

We now turn to firms' incentives to achieve compatibility when transfers between the firms are feasible.

Proposition 7. *Suppose that both firms can make side payments when deciding about compatibility. Then, the following cases emerge:*

i) Firms do not agree on compatibility if under incompatibility one of the firms obtains a monopoly position.

ii) If $\alpha_i^0 \neq 1/2$ and $k < 1/3$, then firms agree on compatibility if under incompatibility market sharing occurs.

iii) If $\alpha_i^0 \neq 1/2$ and $k > 1/3$, then firms do not agree on compatibility if under incompatibility market sharing occurs.

Moreover, if $\alpha_i^0 = 1/2$ or if $k = 1/3$, then firms are indifferent between compatibility and incompatibility if market sharing prevails under incompatibility.

Proof. See Appendix.

Proposition 7 shows that firms cannot do jointly better even when side-payments are possible, if under incompatibility the monopoly equilibrium emerges. Proposition 7, however, also shows that firms may agree on compatibility whenever the market sharing equilibrium holds under incompatibility. Namely, if switching costs are relatively small (or, network effects are sufficiently large) such that $k < 1/3$ holds, then firms can increase their joint profits if side payments are feasible. If, to the contrary, $k > 1/3$ holds, then firms can never jointly do better by making their products compatible.

We are now interested in consumers' preferences concerning compatibility. When network effects are large, then consumers overall expenses are larger than under incompatibility while the opposite holds if switching costs become larger.

Proposition 8. *Consumers are always better off under compatibility when compared with incompatible products.*

Proof. See Appendix.

Proposition 8 shows that consumers are always better off when products are compatible. This result is independent of the type of equilibrium that emerges under incompatibility. Under compatibility network effects are maximized while at the same time switching is more costly for consumers under incompatibility. Comparing firms' rather low incentives to achieve compatibility (except for the instances with $k < 1/3$) we can conclude that consumers' and firms' interests in compatibility are typically not aligned.

We conclude our discussion of firms' compatibility incentives with the comparison of social welfare under both regimes.

Proposition 9. *The comparison of social welfare under compatibility and incompatibility depends on the type of equilibrium under incompatibility.*

Case i). Suppose that under incompatibility the market sharing equilibrium emerges. If $k > 5/6$*, then there exists a unique threshold value,* $\mu_1(k) < \overline{\alpha}^0(k)$*, such that for all* $\alpha_i^0 \in (\mu_1(k), \overline{\alpha}^0(k))$ *and* $\alpha_i^0 \in (\overline{\overline{\alpha}}^0(k), 1 - \mu_1(k))$ *social welfare is strictly larger under incompatibility than under compatibility. In all other cases, social welfare is higher under compatibility (with indifference holding if* $\alpha_i^0 \in \{\mu_1(k), 1 - \mu_1(k)\}$*). Moreover,* $\mu_1(k)$ *is monotonically increasing and it holds that* $\mu_1(5/6) = \overline{\alpha}^0(5/6)$ *and* $\mu_1((103 + \sqrt{1105})/132) = 1$*.*

Case ii) Suppose that under incompatibility the monopoly equilibrium emerges. If $\alpha_i^0 < 1/5$ *(*$\alpha_i^0 > 4/5$*), then social welfare is strictly higher in the monopoly equilibrium where firm* j *(firm* i*) becomes the monopolist (*$i, j = A, B$ *and* $i \neq j$*). In all other instances social welfare is larger under compatibility (with indifference holding if* $\alpha_i^0 \in \{1/5, 4/5\}$*).*

Proof. See Appendix.

Proposition 9 shows that social welfare can be larger under incompatibility than under compatibility. Quite intuitively, the monopoly outcome under incompatibility appears to be attractive if the initial market share of the firm which becomes the monopolist in the equilibrium is already large. In those instances consumers' switching costs are not too large while network effects become maximized in the monopoly outcome. In fact, Proposition 9 states that the monopoly outcome under incompatibility leads to a higher level of social welfare than under compatibility if the prospective monopolist's installed base is larger than four-fifth. In that region the relatively higher level of consumer switching under compatibility makes incompatibility more attractive than compatibility. Finally, Proposition 9 also shows the existence of a (small) parameter range where social welfare is higher in the market sharing equilibrium under incompatibility when compared with compatible products. Again, in that interval the relatively higher switching costs incurred under compatibility in connection with relatively high network effects under incompatibility give rise to the surprising result that social welfare can be higher under incompatibility.

4.3 Incentives to Increase Switching Costs

We now turn to firms' incentives to increase consumer switching costs. We analyze firms' incentives to raise switching at the margin, so that incentives follow from the sign of $\partial\pi/\partial t$.[26] We first analyze incentives in the market sharing equilibrium under incompatibility. In that case firms' profits are given by $t(\alpha_i^0)^2$, so that the direct effect of an increase in switching costs on firms' profits is always positive. However, there is also an indirect effect running through firms' market shares. Taking the derivative of firm i's market share, $\alpha_i^I(\alpha_i^0, k)$, with respect to t yields

$$\frac{\partial\alpha_i^I(\alpha_i^0, k)}{\partial t} = \frac{1-2\alpha_i^0}{b(3k-2)^2}. \tag{15}$$

From (15) it follows immediately that the equilibrium market share of the initially dominant firm in the market sharing equilibrium decreases as t increases, whereas the market share of the other (initially smaller) firm must increase. If market shares are the same initially, then $\alpha_A^I(\alpha_A^0, k) = \alpha_B^I(\alpha_B^0, k)$ holds for any level of the switching costs, t, so that firms' have no strict incentives in that particular case. Obviously, the smaller firm gains strictly from an increase in switching costs, as both the direct and the indirect effects of an increase in switching costs tend to raise its profit. This is not necessarily the case for the initially dominant firm as is stated in the following proposition.

Proposition 10. *Suppose the market sharing equilibrium under incompatibility. Then, the initially smaller firm's profit strictly increases as switching costs increase. For the initially dominant firm, there exists a unique threshold value $\widetilde{\alpha}^0(k) := [3k(1-k)-2]/[3k(k-2)]$ such that the profit of the initially dominant firm increases as switching costs increase if $\alpha_i^0 < \widetilde{\alpha}^0(k)$ holds, while its profit decreases otherwise (with indifference holding if $\alpha_i^0 = \widetilde{\alpha}^0(k)$). The threshold value $\widetilde{\alpha}^0(k)$ is strictly convex with $\partial\widetilde{\alpha}^0(k)/\partial k < 0$ for all $k < 2/3$ and $\partial\widetilde{\alpha}^0(k)/\partial k > 0$ for all $k > 2/3$. Moreover, $\widetilde{\alpha}^0(k) = 1$ if $k \in \{(1/12)(9-\sqrt{33}), (1/12)(9+\sqrt{33})\}$.*

Proof. See Appendix.

Proposition 10 shows that an initially dominant firm has an incentive to raise switching costs if its market share is not large. Interestingly, we obtain that both firms' interests are always

[26] In our analysis we focus on marginal changes of the parameter t (and thus, of parameter k). We, therefore, assume that a change in switching costs does not change the type of equilibrium.

aligned if either network effects are large (so that $k < (1/12)(9 - \sqrt{33})$) or switching costs dominate (such that $k > (1/12)(9 + \sqrt{33})$). In contrast, if switching costs and network effects are more balanced, then a conflict of interests becomes more likely, in particular, whenever firms' installed bases are sufficiently asymmetric.

Let us next examine the incentives to increase switching costs whenever the monopoly equilibrium emerges under incompatibility with $\alpha_i^M = 1$. In that case the profit of the monopolist is given by $\pi_i^M(\alpha_i^0, k) = b\left[1 - k(1 - \alpha_i^0)\right]$ and the profit of the losing rival firm j is zero. The following result is now immediate.

Proposition 11. *Suppose $\alpha_i^0 < 1$ $(i = A, B)$. If the monopoly equilibrium emerges under incompatibility with $\alpha_i^M = 1$, then firm i has no (strict) incentive to raise switching costs, while firm j $(j \neq i)$ is indifferent in that case. If $\alpha_i^0 = 1$, then both firms do neither gain nor lose from a change in switching costs.*

Proposition 11 shows that a prospective monopolist does not have any incentives to increase switching costs, as higher switching costs decrease the equilibrium price. In other words, it is easier to monopolize the market when switching costs are relatively low. Conversely, the losing rival firm finds it increasingly difficult to break consumers' monopolizing expectations the smaller switching costs become. Proposition 11 also shows that a firm which already holds a monopoly position lacks any incentives to increase switching costs further.

We finally state our result concerning firms' incentives to raise switching costs when products are compatible.

Proposition 12. *Under compatibility, both firms always have strict incentives to increase switching costs.*

Proposition 12 follows immediately from firms' profits under compatibility which are given by $t[(1 + \alpha_i^0)/3]^2$ $(i = A, B)$, so that the indirect effect which creates conflicting interests under incompatibility is absent under compatibility. As only the direct effect prevails both firms have always strict incentives to raise switching costs.

Our analysis of firms' incentives to raise switching costs reveals a potentially important drawback under compatibility. As compatibility unambiguously aligns both firms' incentives to raise switching costs, markets with compatible products may end up with overall higher switching costs when compared with markets where products remain incompatible. This observation should be particularly true if the market is monopolized under incompatibility as in that case incentives to raise switching costs are completely absent (see Proposition 11).

5 Conclusion

We presented a model of duopolistic Bertrand competition in a market where both network effects and consumer switching costs shape competitive outcomes. Our main contribution is the analysis of market dynamics when products are incompatible. We showed that in the unique market sharing equilibrium (which always exists) firms' market shares converge either towards market sharing or towards monopolization. Market dynamics are either monotone (in which case the initially dominant firm gradually gains or loses market shares) or alternating (in which case firms interchange dominant positions). The exact type of market dynamics critically depends on the ratio of switching costs to network effects, where small changes of that ratio can have dramatic consequences. Precisely, if network effects dominate switching costs, then market shares converge towards market sharing in an alternating fashion. In the opposite case, when switching costs dominate network effects the market, again, converges towards equal market sharing, but in a monotone way. However, whenever network effects and switching costs are balanced, then market shares always converge towards monopolization either in a monotone or in an alternating course. Our model, therefore, nests previous results derived in the switching costs and network effects literature, respectively, and reveals that the delicate interplay of both market forces gives rise to new results (i.e., when both forces are balanced). In the area where switching costs and network effects are balanced we also obtained a critical mass effect, such that a region of parameter constellations emerges where the initially dominant firm becomes the monopolist for sure at the end of the period (as a result of a unique equilibrium prediction). Interestingly, neither large network effects or large switching only can drive the industry into a monopoly outcome for sure. In the former case the multiplicity of equilibria and in the latter case strict convergence towards market sharing rule out the establishment of an uncontestable monopoly outcome. Taking a longer run perspective, we also analyzed stable and stationary equilibria if our market game is infinitely often repeated and agents are myopic. An important lesson here was that the market is likely to finish in a monopoly outcome if switching costs grow gradually over time.

The comparison of social welfare and consumer surplus under incompatibility in the market sharing equilibrium and the monopoly outcomes (if both coexist) highlights a fundamental trade-off between both policy goals. While the very existence of network effects dictates a monopoly outcome from a social welfare point of view, a market sharing outcome is preferred from a consumer perspective. That result may explain why policy makers taking an industrial policy perspective (and hence, primarily focus on profits) tend to favor picking a winning standard out of incompatible alternatives whereas in competition policy circles (which are supposed to focus on consumer surplus) a more tentative assessment appears to have gained control.

We also analyzed market outcomes when products are compatible. Most importantly, we showed that in contrast to often expressed views concerning the desirability of compatibility social welfare is strictly higher under incompatibility if a prospective monopolist already holds a sufficiently large market share. The reason for this result is that switching costs under compatibility are larger in that case while network effects are maximized under both regimes. Imposing compatibility in a market where a firm already holds a dominant position may, therefore, involve welfare losses which depend on the importance of consumer switching costs.

Finally, we examined incentives to raise switching costs where the main lesson was that under incompatibility firms' interests may not be aligned while under compatibility both firms have strict incentives to increase switching costs so as to lessen competition. Again, that result highlights a possible drawback of promoting compatibility as this may lead to welfare losses caused by higher switching costs in the market.

There are many open questions for further research. One concern is the analysis of market dynamics, when markets grow and consumers can switch technologies. In those instances market dynamics depend on the growth rate and the feasibility of price discrimination strategies. Moreover, dynamics may depend on intertemporal optimization plans and associated dynamic equilibrium behavior. Another important topic for further research should be the analysis of firms' strategies and consumer behavior in markets with strategic uncertainty due to substantial coordination problems. While we have shown that dynamics under incompatibility (i.e., when the coordination problem arises) depend critically on the exact interplay between switching costs and network effects, firms' strategies (as, e.g., product pre-announcements which may focalize consumers' choices) and behavioral rules of consumers (as e.g., preferences for risk dominant strategies) may also determine market outcomes in just the same way.

Appendix

In this Appendix we provide the omitted proofs.

Proof of Proposition 3. From Proposition 1 we know that monopoly equilibria and the market sharing equilibrium coexist if $\alpha_i^0 \in (\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$, with $i = A, B$, which implies $k < 2/3$. We first examine consumer surplus and then turn to social welfare.

Apart from the stand alone value, v, consumer surplus consists of three terms; namely, the value of the network effects, incurred switching costs, and consumers' overall expenses.[27] In the market sharing equilibrium those terms are given by $b[\left(\alpha_i^I\right)^2 + \left(1-\alpha_i^I\right)^2]$, $(1/2)\left(\alpha_i^I - \alpha_i^0\right)(b - t)\left(2\alpha_i^I - 1\right)$, and $t[(\alpha_i^I)^2 + (1-\alpha_i^I)^2]$, respectively (for $i = A, B$). Adding all three terms we can (implicitly) express consumer surplus in the market sharing equilibrium as

$$\frac{CS^I(\alpha_i^I, k) - v}{b} = (1-k)\left[2\left(\alpha_i^I\right)^2 - 2\alpha_i^I + 1 - \frac{1}{2}(\alpha_i^I - \alpha_i^0)(2\alpha_i^I - 1)\right]. \tag{16}$$

Substituting $\alpha_i^I(k, \alpha_i^0)$ (given by (7)) into (16) we obtain

$$\frac{CS^I(\alpha_i^0, k) - v}{b} = \frac{(1-k)\left[4k(1-2k)\alpha_i^0(1-\alpha_i^0) + 11k^2 - 13k + 4\right]}{18(k-2/3)^2}. \tag{17}$$

In the monopoly equilibrium with firm i ($i = A, B$) gaining the entire market, consumer surplus is given by $CS_i^M(\alpha_i^0, k) = v + (t/2)\left[1 - (\alpha_i^0)^2\right]$ which we can re-write as $[CS_i^M(\alpha_i^0, k) - v]/b = (k/2)\left[1 - (\alpha_i^0)^2\right]$. Thus, the comparison of consumers' surpluses under the market sharing and the monopoly equilibrium gives rise to the following expression

$$\frac{CS^I(\alpha_i^0, k) - CS_i^M(\alpha_i^0, k)}{b} = \frac{k^3\left[\alpha_i^0 - (2k-1)/k\right]\left[\alpha_i^0 - \left[k(13-10k) - 4\right]/k^2\right]}{2\left(3k-2\right)^2}. \tag{18}$$

Defining $\widehat{\alpha}^0(k) := \left[k(13-10k) - 4\right]/k^2$ and substituting $\widehat{\alpha}^0(k)$ and $\overline{\alpha}^0(k) := (2k-1)/k$ into the right-hand side of Equation (18) we obtain

$$\frac{CS^I(\alpha_i^0, k) - CS_i^M(\alpha_i^0, k)}{b} = \frac{k^3}{2\left(3k-2\right)^2}\left[\alpha_i^0 - \overline{\alpha}^0(k)\right]\left[\alpha_i^0 - \widehat{\alpha}^0(k)\right]. \tag{19}$$

For the case that firm j ($j = A, B$, $j \neq i$) becomes the monopolist in the monopoly equilibrium we obtain the following expression (which follows from replacing $\overline{\alpha}^0(k)$ by $\overline{\overline{\alpha}}^0(k)$ and $\widehat{\alpha}^0(k)$ by

[27] Switching costs follow from the formula $(1/2)(\alpha_A^* - \alpha_A^0)(U_{x=0}^A - U_{x=1}^B)$.

$1 - \widehat{\alpha}^0(k)$ in (19))

$$\frac{CS^I(\alpha_i^0, k) - CS_j^M(\alpha_i^0, k)}{b} = \frac{k^3}{2\,(3k-2)^2}\left[\alpha_i^0 - \overline{\overline{\alpha}}^0(k)\right]\left[\alpha_i^0 - (1 - \widehat{\alpha}^0(k))\right]. \tag{20}$$

From Equation (19) we observe that the sign of $CS^I(\alpha_i^0, k) - CS^M(\alpha_i^0, k)$ is determined by the sign of $\left[\alpha_i^0 - \overline{\alpha}^0(k)\right][\alpha_i^0 - \widehat{\alpha}^0(k)]$. Let us now examine the properties of $\widehat{\alpha}^0(k)$ and how it is related to $\overline{\alpha}^0(k)$ and $\overline{\overline{\alpha}}^0(k)$. Successive differentiation of $\widehat{\alpha}^0(k)$ yields $\partial\widehat{\alpha}^0/\partial k = -(13k-8)/k^3$ and $\partial^2\widehat{\alpha}^0/\partial k^2 = [2(13k-12)]/k^4$. Note that $\partial^2\widehat{\alpha}^0/\partial k^2 < 0$ for all $k < 2/3$. Hence, $\widehat{\alpha}^0(k)$ is strictly concave over $k \in (0, 2/3)$ and obtains a unique maximum at $k = 8/13$ with $\widehat{\alpha}^0(8/13) = 9/16$. Note further that $\widehat{\alpha}^0(1/2) = 0$. As $\overline{\alpha}^0(k)$ is strictly increasing over $k \in (0, 2/3)$ and obtains a zero at $k = 1/2$, we know that $\widehat{\alpha}^0(k)$ and $\overline{\alpha}^0(k)$ are nonpositive for all $k \leq 1/2$. Hence for all $k \leq 1/2$ the right-hand side of (19) must be strictly positive as well, so that consumer surplus is larger in the market sharing equilibrium for any $\alpha_i^0 > 0$, when compared with the monopoly equilibrium.

Turning to the comparison of consumer surplus when firm j ($j \neq i$) becomes the monopolist (see Equation (20)), we first notice that $(1 - \widehat{\alpha}^0(k))$ is the exact mirror image of $\widehat{\alpha}^0(k)$, so that $(1 - \widehat{\alpha}^0(k))$ is strictly convex over $k \in (1/2, 2/3)$, reaches a unique minimum at $k = 8/13$ with $(1 - \widehat{\alpha}^0(8/13)) = 7/16$, and obtains the values $(1 - \widehat{\alpha}^0(1/2)) = 0$ and $(1 - \widehat{\alpha}^0(2/3)) = 1/2$. Moreover, $\widehat{\alpha}^0(k) = (1 - \widehat{\alpha}^0(k))$ at $k = 4/7$ and $\lim_{k\to 2/3}\widehat{\alpha}^0(k) = \lim_{k\to 2/3}(1 - \widehat{\alpha}^0(k)) = 1/2$. Inspecting (20) we then obtain that $[\alpha_i^0 - \overline{\overline{\alpha}}^0(k)]$ and $[\alpha_i^0 - (1 - \widehat{\alpha}^0(k))]$ are strictly negative for all $\alpha_i^0 > 0$ if $k \leq 1/2$. Hence, consumer surplus is always larger in the monopoly equilibrium where firm j becomes the monopolist when compared with the market sharing equilibrium. This proves part i) of Proposition 3.

In the interval $k \in (1/2, 2/3)$ multiple equilibria emerge only if $\alpha_i^0 \in (\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$. We first focus on the case when firm i becomes the monopolist where the comparison of consumer surplus depends on Equation (19). We have to analyze how $\widehat{\alpha}^0(k)$ is related to $\overline{\alpha}^0(k)$ and $\overline{\overline{\alpha}}^0(k)$ in the interval $k \in (1/2, 2/3)$. The following claim shows that $\widehat{\alpha}^0(k)$ lies exactly between the upper boundary, $\overline{\overline{\alpha}}^0(k)$, and the lower boundary, $\overline{\alpha}^0(k)$.

Claim 1. *$\widehat{\alpha}^0(k) - \overline{\alpha}^0(k) > 0$ and $\overline{\overline{\alpha}}^0(k) - \widehat{\alpha}^0(k) > 0$ hold for all $k \in (1/2, 2/3)$.*

Proof. Simple calculation give $\widehat{\alpha}^0(k) - \overline{\alpha}^0(k) = 12(1/2 - k)(k - 2/3)/k^2$ which is clearly strictly positive over the interval $k \in (1/2, 2/3)$. Similarly, we obtain $\overline{\overline{\alpha}}^0(k) - \widehat{\alpha}^0(k) = (3k-2)^2/k^2$

which is obviously strictly positive. This proves Claim 1.

From Claim 1 we know that α_i^0 lies either in the interval $(\overline{\alpha}^0(k), \widehat{\alpha}^0(k))$ or in the interval $(\widehat{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$. In the former case $\left[\alpha_i^0 - \overline{\alpha}^0(k)\right] > 0$ and $[\alpha_i^0 - \widehat{\alpha}^0(k)] < 0$, so that the right-hand side of Equation (19) is strictly negative. Hence, consumer surplus is higher in the monopoly equilibrium if $\alpha_i^0 \in (\overline{\alpha}^0(k), \widehat{\alpha}^0(k))$ for $k \in (1/2, 2/3)$. Consider now the remaining case with $\alpha_i^0 \in (\widehat{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$, where $\left[\alpha_i^0 - \overline{\alpha}^0(k)\right] > 0$ and $[\alpha_i^0 - \widehat{\alpha}^0(k)] > 0$, so that the right-hand side of Equation (19) is strictly positive for all $k \in (1/2, 2/3)$ and consumer surplus, therefore, is strictly larger in the market sharing equilibrium when compared with the monopoly equilibrium when firm i becomes the monopolist.

We now turn to the case where firm j $(j \neq i)$ becomes the monopolist in the monopoly equilibrium in which case the comparison depends on Equation (20). It is immediate from Claim 1 that $(1 - \widehat{\alpha}^0(k)) - \overline{\overline{\alpha}}^0(k) < 0$ and $\overline{\alpha}^0(k) - (1 - \widehat{\alpha}^0(k)) < 0$ hold for all $k \in (1/2, 2/3)$. Inspecting (20) we observe that $[\alpha_i^0 - \overline{\overline{\alpha}}^0(k)] < 0$ must always hold, so that consumer surplus is larger in the market sharing equilibrium than in the monopoly equilibrium with firm j becoming the monopolist if and only if $[\alpha_i^0 - (1 - \widehat{\alpha}^0(k))] < 0$ or $\alpha_i^0 < 1 - \widehat{\alpha}^0(k)$ is fulfilled. This proves part ii) of Proposition 3. Part iii) follows from combining the results derived in part ii).

We turn now to the comparison of social welfare. Social welfare is given by the sum of consumer surplus and firms' profits, where the latter is given by consumers' overall expenses, $t[(\alpha_i^I)^2 + (1 - \alpha_i^I)^2]$. Adding firms' profits to (16) we can express social welfare in the market sharing equilibrium implicitly as

$$\frac{SW^I(\alpha_i^I, k) - v}{b} = 2\left(\alpha_i^I\right)^2 - 2\alpha_i^I + 1 - \frac{1}{2}(1-k)\left(\alpha_i^I - \alpha_i^0\right)(2\alpha_i^I - 1). \tag{21}$$

Substituting $\alpha_i^I(k, \alpha_i^0)$ (given by (7)) into (21) yields

$$\frac{SW^I(\alpha_i^0, k) - v}{b} = \frac{4k\alpha_i^0\left(\alpha_i^0 - 1\right)[k(3-k) - 1] - k^3 + 12k^2 - 13k + 4}{2\left(3k-2\right)^2}. \tag{22}$$

Accordingly, we can express social welfare in the monopoly equilibrium when firm i becomes the monopolist as[28]

$$\frac{SW_i^M(\alpha_i^0, k) - v}{b} = 1 - \frac{1}{2}k\left(1 - \alpha_i^0\right)^2. \tag{23}$$

[28] We omit the proof for the case where firm j $(j \neq i)$ becomes the monopolist in the monopoly equilibrium which proceeds analogously.

Using (22) and (23) the difference between social welfare in the market sharing and the monopoly equilibrium is (implicitly) given by

$$\frac{SW^I(\alpha_i^0,k)-SW_i^M(\alpha_i^0,k)}{b}=\frac{5k^3}{2\left(3k-2\right)^2}\left[\alpha_i^0-\frac{2k-1}{k}\right]\left[\alpha_i^0-\frac{k(4k-7)+4}{5k^2}\right]. \tag{24}$$

Defining $\phi(k):=\left[k(4k-7)+4\right]/\left(5k^2\right)$ and substituting $\phi(k)$ and $\overline{\alpha}^0(k):=\left(2k-1\right)/k$ into the right-hand side of Equation (24) we obtain

$$\frac{SW^I(\alpha_i^0,k)-SW_i^M(\alpha_i^0,k)}{b}=\frac{5k^3}{2\left(3k-2\right)^2}\left[\alpha_i^0-\overline{\alpha}^0(k)\right]\left[\alpha_i^0-\phi(k)\right]. \tag{25}$$

From Equation (25) we observe that the sign of $SW^I(\alpha_i^0,k)-SW_i^M(\alpha_i^0,k)$ is determined by the sign of $\left[\alpha_i^0-\overline{\alpha}^0(k)\right]\left[\alpha_i^0-\phi(k)\right]$. Let us now examine the properties of $\phi(k)$ and how it is related to $\overline{\alpha}^0(k)$. Note first that $\partial\phi/\partial k=(7k-8)/(5k^3)$, from which we see directly that $\phi(k)$ is strictly decreasing over the interval $k\in(0,2/3)$. As $\phi(1/2)=6/5>1$ holds we know that $[\alpha_i^0-\phi(k)]<0$ must hold for all $k\in(0,1/2]$. As $\overline{\alpha}^0(k)\leq 0$ holds for all $k\in(0,1/2]$ we know that $\left[\alpha_i^0-\overline{\alpha}^0(k)\right]>0$ must be true over that interval (note that $\alpha_i^0>0$). Hence, the right-hand side of Equation (25) is strictly negative over the interval $k\in(0,1/2]$ which implies that social welfare is higher in the monopoly equilibrium when compared with the market sharing equilibrium.

We now turn to the analysis of the remaining interval $k\in(1/2,2/3)$, where the market sharing equilibrium only exists if $\alpha_i^0\in(\overline{\alpha}^0(k),\overline{\overline{\alpha}}^0(k))$. As in the first part of the proof we are interested how $\phi(k)$ is related to $\overline{\alpha}^0(k)$ and $\overline{\overline{\alpha}}^0(k)$. The next claim shows that $\phi(k)>\overline{\overline{\alpha}}^0(k)$, so that $[\alpha_i^0-\phi(k)]<0$ must hold for all $k\in(1/2,2/3)$.

Claim 2. *$\phi(k)-\overline{\overline{\alpha}}^0(k)>0$ holds for all $k\in(1/2,2/3)$.*

Proof. The difference $\phi(k)-\overline{\overline{\alpha}}^0(k)$ can be rewritten as $\phi(k)-\overline{\overline{\alpha}}^0(k)=\left(3k-2\right)^2/(5k^2)$ which is clearly strictly positive over the interval $k\in(1/2,2/3)$. This proves Claim 2.

With Claim 2 at hand we know that for any α_i^0 for which both market sharing and monopoly equilibria emerge, i.e., $\alpha_i^0\in(\overline{\alpha}^0(k),\overline{\overline{\alpha}}^0(k))$, it holds that $[\alpha_i^0-\phi(k)]<0$. As $\alpha_i^0>\overline{\alpha}^0(k)$ must hold to ensure that both monopoly equilibria and the market sharing equilibrium coexist, we know that $\left[\alpha_i^0-\overline{\alpha}^0(k)\right]>0$ must hold for all $k\in(1/2,2/3)$. Hence, the right-hand side of Equation (25) is strictly negative for all $k\in(1/2,2/3)$ which completes the proof of Proposition 3. *Q.E.D.*

Proof of Proposition 4. Consider first consumer surplus in the market sharing equilibrium which is given by (17). We have to compare consumer surplus for firms' initial market shares α_i^0 with consumer surplus in the outcome in the market sharing equilibrium $\alpha_i^I(\alpha_i^0, k)$ (given by (7)). Without loss of generality let $\alpha_i^0 > 1/2$. For the sake of brevity define $\alpha_i^0 := \alpha_1$ and $\alpha_i^I := \alpha_2$, so that the difference between $CS^I(\alpha_2, k)$ and $CS^I(\alpha_1, k)$ can be stated as

$$\frac{CS^I(\alpha_2, k) - CS^I(\alpha_1, k)}{b} = \frac{2k(1-k)(1-2k)\left[\omega(\alpha_2) - \omega(\alpha_1)\right]}{(3k-2)^2}, \tag{26}$$

where $\omega(\alpha) := \alpha(1-\alpha)$. Clearly, the function $\omega(\alpha)$ is maximized at $\alpha = 1/2$ and symmetric around that point. Before proceeding with the inspection of (26) it is useful to specify the properties of the function $\omega(\alpha)$ which allows us to specify the sign of the difference $\omega(\alpha_2) - \omega(\alpha_1)$.

Claim 3. *Assume* $\alpha_1 > 1/2$.

i) If $\alpha_2 > 1/2$, *then* $\omega(\alpha_1) > \omega(\alpha_2)$ *for* $\alpha_1 < \alpha_2$ *and* $\omega(\alpha_1) < \omega(\alpha_2)$ *for* $\alpha_1 > \alpha_2$.

ii) If $\alpha_2 < 1/2$, *then* $\omega(\alpha_1) > \omega(\alpha_2)$ *for* $\alpha_1 - 1/2 < 1/2 - \alpha_2$ *and* $\omega(\alpha_1) < \omega(\alpha_2)$ *for* $\alpha_1 - 1/2 > 1/2 - \alpha_2$.

Proof. First notice that $\partial\omega(\alpha)/\partial\alpha = 1 - 2\alpha$. Hence, the function increases for all $\alpha < 1/2$, decreases for all $\alpha > 1/2$ and reaches its maximum at $\alpha = 1/2$. Then the part i) of the Claim 1 is immediate.

For part ii) note next that $\omega(\alpha)$ is symmetric around $\alpha = 1/2$; i.e., for any x it holds that $\omega(1/2 - x) = \omega(1/2 + x)$. For any $x, y > 0$ it then follows that $\omega(1/2 + x) > \omega(1/2 - y)$ if $x < y$ and $\omega(1/2 + x) < \omega(1/2 - y)$ if $x > y$. To show this, assume $x > y$. Using the symmetry property we get $\omega(1/2 + x) = \omega(1/2 - x)$. As $\omega(\alpha)$ is strictly increasing for all $\alpha < 1/2$ we obtain $\omega(1/2 - x) < \omega(1/2 - y)$ as $1/2 - x < 1/2 - y$ and, hence, $\omega(1/2 + x) = \omega(1/2 - x) < \omega(1/2 - y)$. As $1/2 + x > 1/2$ and $1/2 - y < 1/2$, we can set $1/2 + x = \alpha_1$ and $1/2 - y = \alpha_2$, so that $x > y$ is equivalent with $\alpha_1 - 1/2 > 1/2 - \alpha_2$ and $\omega(1/2 + x) < \omega(1/2 - y)$ is equivalent with $\omega(\alpha_1) < \omega(\alpha_2)$. This proves the claim.

With the properties of the function $\omega(\alpha)$ at hand, we consider now all possible market dynamics as specified in Proposition 2. Recall also that we assume $\alpha_1 > 1/2$. Consider first $k > 1$, for which according to Proposition 2 we have $\alpha_2 > 1/2$ and $\alpha_1 > \alpha_2$. Hence, $\omega(\alpha_1) < \omega(\alpha_2)$, so that the difference $\omega(\alpha_2) - \omega(\alpha_1)$ of the right-hand side in (26) is strictly positive. As $(1-k) < 0$ and $(1-2k) < 0$ must hold for $k > 1$, it follows that (26) is positive. Hence, consumer surplus increases for $k > 1$. Consider now $2/3 < k < 1$, for which according to Proposition 2 $\alpha_2 > 1/2$ and $\alpha_1 < \alpha_2$. Hence, $\omega(\alpha_1) > \omega(\alpha_2)$ and the difference $\omega(\alpha_2) - \omega(\alpha_1)$ of the right-hand side in (26) is negative. As $(1-k) > 0$ and $(1-2k) < 0$ hold for all $2/3 < k < 1$, (26) is also positive, so that consumer surplus increases in that region. Consider now $1/2 < k < 2/3$, for which according to Proposition 2 $\alpha_2 < 1/2$ and $\alpha_1 - 1/2 < 1/2 - \alpha_2$. Hence, $\omega(\alpha_1) > \omega(\alpha_2)$ and the difference $\omega(\alpha_2) - \omega(\alpha_1)$ of the right-hand side in (26) is negative. As $(1-k) > 0$ and $(1-2k) < 0$, the right-hand side of (26) is positive as well. Hence, consumer surplus increases for all $1/2 < k < 2/3$. Consider finally $k < 1/2$ for which according to Proposition 2 $\alpha_2 < 1/2$ and $\alpha_1 - 1/2 > 1/2 - \alpha_2$. Hence, $\omega(\alpha_1) < \omega(\alpha_2)$ and the difference $\omega(\alpha_2) - \omega(\alpha_1)$ on the right-hand side of (26) is positive. As $(1-k) > 0$ and $(1-2k) > 0$ hold in that area, the right-hand side of (26) is a positive value. Hence, consumer surplus increases for all $k < 1/2$. Note also that $CS^I(\alpha_2, k) = CS^I(\alpha_1, k)$ if either $k = 1/2$, or $k = 1$ or if $\omega(\alpha_2) = \omega(\alpha_1)$. Due to the symmetry of the function $\omega(\alpha)$ the latter holds for $\alpha_2 = \alpha_1$ and $\alpha_1 - 1/2 = 1/2 - \alpha_2$, what is equivalent to $\alpha_1 = 1 - \alpha_2$. Solving $\alpha_i^0 = \alpha_i^I(\alpha_i^0, k)$ we get $k = 1$ and $\alpha_i^0 = 1/2$, solving $\alpha_i^0 = 1 - \alpha_i^I(\alpha_i^0, k)$ we get $k = 1/2$ and $\alpha_i^0 = 1/2$. Hence, if $\alpha_i^0 = 1/2$ or if $k = 1/2$ or $k = 1$ hold, then the right-hand side of (26) is zero.

Let us now consider how social welfare changes in the market sharing equilibrium. We compare again the values of the function $SW^I(\alpha, k)$ at $\alpha = \alpha_1$ and $\alpha = \alpha_2$. We again assume $\alpha_1 > 1/2$. The difference between $SW^I(\alpha_2, k)$ and $SW^I(\alpha_1, k)$ is given by

$$\frac{SW^I(\alpha_2, k) - SW^I(\alpha_1, k)}{b} = \frac{4k\left[k - \frac{(3-\sqrt{5})}{2}\right]\left[k - \frac{(3+\sqrt{5})}{2}\right][\omega(\alpha_2) - \omega(\alpha_1)]}{2\,(3k-2)^2} \tag{27}$$

Inspecting (27), note first that $(3+\sqrt{5})/2 > 1$ and $(3-\sqrt{5})/2 < 1/2$. Let us define the first two terms in rectangular brackets on the right-hand side in (27) by $\zeta(k)$. Hence, $\zeta(k)$ is positive whenever $k > (3+\sqrt{5})/2$ and $k < (3-\sqrt{5})/2$ hold and obtains negative values otherwise. Hence, if $k > (3+\sqrt{5})/2$, then the right-hand side of (27) is positive as the difference $\omega(\alpha_2) - \omega(\alpha_1)$ is also positive in that region. For $1 < k < (3+\sqrt{5})/2$ the difference $\omega(\alpha_2) - \omega(\alpha_1)$ is still positive, but $\zeta(k)$ takes negative values here; hence, the right-hand side of (27) is negative, so that social welfare decreases in the market sharing equilibrium if $1 < k < (3+\sqrt{5})/2$. Consider now the range $2/3 < k < 1$, for which the difference $\omega(\alpha_2) - \omega(\alpha_1)$ is negative and $\zeta(k)$ takes again negative values. Hence, the right-hand side of (27) is positive in that area. For

$1/2 < k < 2/3$ both the difference $\omega(\alpha_2) - \omega(\alpha_1)$ and $\zeta(k)$ are negative, so that the right-hand side of (27) is positive in that region. For $(3-\sqrt{5})/2 < k < 1/2$ the difference $\omega(\alpha_2) - \omega(\alpha_1)$ becomes positive and $\zeta(k)$ takes negative values, so that the right-hand side of (27) is also negative. Consider finally $0 < k < (3-\sqrt{5})/2$, for which both the difference $\omega(\alpha_2) - \omega(\alpha_1)$ and $\zeta(k)$ are positive, so that the right-hand side of (27) obtains positive values. Note finally that $SW^I(\alpha_2, k) - SW^I(\alpha_1, k) = 0$ if either $\zeta(k)$ or the difference $\omega(\alpha_2) - \omega(\alpha_1)$ is zero. Hence, social welfare does not change if either $\alpha_i^0 = 1/2$ or if $k \in \{(3-\sqrt{5})/2, 1/2, 1, (3+\sqrt{5})/2\}$. This completes the proof of the proposition. *Q.E.D.*

Proof of Lemma 3. First we rule out the existence of a monopoly equilibrium. We proceed by contradiction. Assume that in the monopoly equilibrium $\alpha_i(p_i, p_j; \alpha_i^0) = 1$ (with $i, j = A, B$, $j \neq i$). It must then hold that $p_j = 0$, as otherwise (with $p_j > 0$) firm j could increase its profit by decreasing its price. From firms' demands (14) it follows that $-p_i \geq t(1 - \alpha_i^0)$ must hold what is only feasible if $p_i = 0$ and $\alpha_i^0 = 1$. In a monopoly equilibrium, firm i must not have an incentive to increase its price above $p_i = 0$. By increasing its price firm i faces the demand given by $\alpha_i^c = \alpha_i^0 + (p_j - p_i)/t$, so that $\partial \pi_i^c(p_i, p_j; \alpha_i^0)/\partial p_i = \alpha_i^0 + (p_j - 2p_i)/t$. Evaluating the derivative at $p_A = p_B = 0$ and $\alpha_i^0 = 1$ we obtain $\partial \pi_i^c / \partial p_i = 1$. Hence, the monopoly outcome cannot be an equilibrium under compatibility.

In the market sharing equilibrium firm i's demand is given by $\alpha_i^c(p_i, p_j; \alpha_i^0) = \alpha_i^0 + (p_j - p_i)/t$, with $i, j = A, B$ and $i \neq j$. Solving firms optimization problems (which are globally concave) we obtain the prices and market shares as unique market sharing equilibrium outcomes as stated in the lemma. The last part of the lemma follows from the fact that $\alpha_i^c(\alpha_i^0) = (\alpha_i^0 + 1)/3 > 1/2$ and $\alpha_i^c(\alpha_i^0) = (\alpha_i^0 + 1)/3 < \alpha_i^0$ hold for all $\alpha_i^0 > 1/2$. Hence, we obtain monotone market sharing as the unique market dynamic when products are compatible. *Q.E.D.*

Proof of Proposition 6. *Case i).* In the monopoly equilibrium under incompatibility the profit of the monopolist (say, firm $i = A, B$) is given by $\pi_i^M(\alpha_i^0) = b - t(1 - \alpha_i^0)$ and the profit of the rival firm is given by $\pi_j^M(\alpha_j^0) = 0$, with $j \neq i$. Clearly, firm j gains from compatibility as $\pi_j^c(\alpha_j^0) = t(1 + \alpha_j^0)^2/9 > 0$ holds. For the monopolist under incompatibility (firm i) we have to compare $\pi_i^c(\alpha_i^0) = t(1 + \alpha_i^0)^2/9$ and $\pi_i^M(\alpha_i^0) = b - t(1 - \alpha_i^0)$. Comparison of the profits reveals that $\pi_i^c(\alpha_i^0) < \pi_i^M(\alpha_i^0)$ is true if and only if $\varphi_1(\alpha_i^0) < 9/k$ with $\varphi_1(\alpha_i^0) := (\alpha_i^0 - 2)(\alpha_i^0 - 5)$. Note that $\varphi_1' < 0$ for all $\alpha_i^0 \in [0, 1]$. We now analyze different values of k for which the monopoly equilibrium emerges. Consider first $k < 2/3$. If $k < 2/3$, then $9/k > 27/2$. As $\varphi_1(\cdot)$ obtains its maximum at $\alpha_i^0 = 0$ we obtain $\varphi_1(0) = 10 < 27/2$, so that $\pi_i^c(\alpha_i^0) < \pi_i^M(\alpha_i^0)$ must hold for any α_i^0 if $k < 2/3$.

Consider next the interval $2/3 < k \leq 1$. In that region, the monopoly equilibrium only emerges for firm i if α_i^0 fulfills $\alpha_i^0 \in [\overline{\alpha}^0(k), 1]$, with $\overline{\alpha}^0(k) := 2 - 1/k \leq \alpha_i^0 \leq 1$. Note that $2 - 1/k > 1/2$ for any $2/3 < k \leq 1$. Hence, for $2/3 < k \leq 1$ it follows that $\alpha_i^0 > 1/2$. As $\varphi_1(\cdot)$ monotonically decreases over the interval $\alpha_i^0 \in [0, 1]$, we have to show that $\varphi_1(1/2) < 9/k$ for $2/3 < k \leq 1$ which proves that $\pi_i^c(\alpha_i^0) < \pi_i^M(\alpha_i^0)$ holds for any α_i^0 (for which the monopoly equilibrium emerges under $2/3 < k \leq 1$). In fact, evaluating $\varphi_1(\cdot)$ at the point $\alpha_i^0 = 1/2$ we get $\varphi_1(1/2) = 27/4 < 9/k$ if $2/3 < k \leq 1$. Hence for any α_i^0 (for which the monopoly equilibrium emerges under $2/3 < k \leq 1$) it holds that $\pi_i^c(\alpha_i^0) < \pi_i^M(\alpha_i^0)$.

Finally, if $k > 1$ a monopoly equilibrium does not exist. Hence, we have proven part i) of the proposition.

Case ii). In the market sharing equilibrium under incompatibility firm i's profit is given by $t(\alpha_i^I)^2$ and under compatibility by $t(\alpha_i^c)^2$. It is then straightforward that $\pi_i^c - \pi_i^I = t(\alpha_i^c - \alpha_i^I)(\alpha_i^c + \alpha_i^I)$. Hence, the sign of the difference $\pi_i^c - \pi_i^I$ is given by the sign of $\alpha_i^c - \alpha_i^I = (1 - 2\alpha_i^0)/[3(3k-2)]$. It is now easily checked that $\alpha_i^c - \alpha_i^I < 0$ holds if either $k < 2/3$ and $\alpha_i^0 < 1/2$ or $k > 2/3$ and $\alpha_i^0 > 1/2$, while in the remaining cases $\alpha_i^c - \alpha_i^I > 0$ holds. If $\alpha_i^0 = 1/2$, then $\pi_i^c = \pi_i^I$. *Q.E.D.*

Proof of Proposition 7. *Case i)* We first analyze the incentives for compatibility when under incompatibility firm i ($i = A, B$) obtains a monopoly position in equilibrium. In this case we have to compare the sum of firms' profits in the monopoly equilibrium under incompatibility $\sum_{j=A,B} \pi_j^M$ with the sum of firms' profits under compatibility $\sum_{j=A,B} \pi_j^c$, which are given by $b - t(1 - \alpha_i^0)$ and $(t/9)[(1 + \alpha_i^0)^2 + (2 - \alpha_i^0)^2]$, respectively. The sign of the difference $\sum_{j=A,B} \pi_j^c - \sum_{j=A,B} \pi_j^M$ is given by the sign of the expression $\psi_1(\alpha_i^0) - 9/k$, with $\psi_1(\alpha_i^0) := 2(\alpha_i^0 - 7/2)(\alpha_i^0 - 2)$. The function $\psi_1(\cdot)$ is monotonically decreasing over the interval $\alpha_i^0 \in [0, 1]$, and obtains its maximum at $\alpha_i^0 = 0$ with $\psi_1(0) = 14$ and its minimum at $\alpha_i^0 = 1$ with $\psi_1(1) = 5$. Hence, the range of possible values of the function $\psi_1(\cdot)$ is given by $5 \leq \psi_1(\cdot) \leq 14$. From the latter it is straightforward to conclude that for $k \leq 9/14$ (for which $9/k \geq 14$) it holds that $\psi_1(\cdot) - 9/k \leq 0$ for any α_i^0, so that $\sum_{j=A,B} \pi_j^c - \sum_{j=A,B} \pi_j^M \leq 0$, what implies that compatibility is not jointly optimal. The values $k > 1$ are irrelevant since for $k > 1$ no monopoly equilibrium under incompatibility emerges.

Thus it is left to consider $9/14 \leq k < 1$. Then the sign of $\psi_1(\cdot) - 9/k$ depends on the initial market share of firm i, α_i^0, which becomes the monopolist under incompatibility. Inspecting the difference $\psi_2 := \psi_1(\cdot) - 9/k$ we obtain two zeros: $\psi_2^1(k) := 11/4 - (3/4)\sqrt{(k+8)/k}$ and $\psi_2^2(k) := 11/4 + (3/4)\sqrt{(k+8)/k}$. It is straightforward that $\psi_1^2(\cdot) > 1$ for any k. We next show that $0 < \psi_2^1(\cdot) < 1$. Note that $\psi_2^1(\cdot)$ is strictly increasing in k. At $k = 9/14$ we obtain $\psi_2^1(9/14) = 0$ and for $k = 1$ we obtain $\psi_2^1(1) = 1/2$. As we know that the monopoly equilibrium can emerge for firm i only if $\alpha_i^0 \geq \overline{\alpha}^0(k) = 2 - 1/k$, we have to check whether $\psi_2^1(\cdot) > \overline{\alpha}^0(\cdot)$ or $\psi_2^1(\cdot) < \overline{\alpha}^0(\cdot)$ holds. We next show that $\psi_2^1(\cdot) < \overline{\alpha}^0(\cdot)$ holds for $k > 1/3$ and $\psi_2^1(\cdot) \geq \overline{\alpha}^0(\cdot)$ holds for $k \leq 1/3$. In fact, we obtain that $11/4 - (3/4)\sqrt{(k+8)/k} < 2 - 1/k$ holds if $3/4 + 1/k < (3/4)\sqrt{(k+8)/k}$ which is equivalent to $(3/4 + 1/k)^2 < (9/16)(k+8)/k$ or $9/16 + 3/(2k) + 1/k^2 < 9/16 + 9/(2k)$ which is equivalent to $1/k < 3$ and thus $k > 1/3$. Hence, for $9/14 \leq k < 1$, it holds that $\psi_2^1(k) < \overline{\alpha}^0(\cdot)$. Thus, for any α_i^0 for which the monopoly equilibrium emerges it holds that $\alpha_i^0 \in (\psi_2^1(\cdot), 1]$. Note that for any $\alpha_i^0 \in (\psi_2^1(\cdot), 1]$ the function ψ_2 is nonpositive. Hence, $\psi_1(\cdot) - 9/k \leq 0$ and thus $\sum_{j=A,B} \pi_j^c - \sum_{j=A,B} \pi_j^M \leq 0$. We have, therefore, shown that for any k and α_i^0 for which the monopoly equilibrium emerges under incompatibility it holds that $\sum_{j=A,B} \pi_j^c - \sum_{j=A,B} \pi_j^M \leq 0$, which implies that both firms never agree on compatibility. Finally, as $\sum_{j=A,B} \pi_j^c - \sum_{j=A,B} \pi_j^M \leq 0$ holds for any α_i^0 when firm i obtains the monopoly position, then because of the symmetry it follows that the inequality also holds if firm j ($j \neq i$) becomes the monopolist under incompatibility.

Cases ii) and iii). We now analyze the possibility for compatibility when otherwise (under incompatibility) firms would share the market in the equilibrium. The sum of the firms' profits under incompatibility in the market sharing equilibrium is given by

$$\sum_{j=A,B} \pi_j^I = \frac{t\left[2k^2(\alpha_i^0)^2 - 2k^2\alpha_i^0 + 5k^2 - 6k + 2\right]}{(3k-2)^2}$$

and the sum of the firms' profits under compatibility is given by

$$\sum_{j=A,B} \pi_j^c = \frac{t\left[2(\alpha_i^0)^2 - 2\alpha_i^0 + 5\right]}{9}.$$

Then the difference of firms' joint profits under compatibility and incompatibility is given by

$$\sum_{j=A,B} \pi_j^c - \sum_{j=A,B} \pi_j^I = \frac{2(1-3k)(2\alpha_i^0 - 1)^2}{9(3k-2)^2}.$$

Obviously, that difference is positive if $k < 1/3$ and negative if $k > 1/3$ (with equality holding at $k = 1/3$ or $\alpha_i^0 = 1/2$). *Q.E.D.*

Proof of Proposition 8. We start with the comparison of consumer surplus when under incompatibility the market sharing equilibrium emerges (*Case i*) and then proceed with the comparison of consumer surplus when under incompatibility the monopoly equilibrium holds (*Case ii*).

Case i). Assume that under incompatibility the market sharing equilibrium emerges. We proceed by comparing consumer surplus under compatibility and incompatibility. Apart from the stand alone value, v, consumer surplus consists of three terms; namely, the value of the network effects, incurred switching costs, and consumers' overall expenses. In the market sharing equilibrium under compatibility those terms are given by b, $(1/2)t\left(\alpha_i^c - \alpha_i^0\right)(1 - 2\alpha_i^c)$, and $t[(\alpha_i^c)^2 + (1 - \alpha_i^c)^2]$, respectively (for $i = A, B$), so that consumer surplus under compatibility $CS^c(\alpha_i^c, k)$ can be (implicitly) expressed as

$$\frac{CS^c(\alpha_i^c, k) - v}{b} = 1 - \frac{k\left(\alpha_i^c - \alpha_i^0\right)(1 - 2\alpha_i^c)}{2} - k[(\alpha_i^c)^2 + (1 - \alpha_i^c)^2]. \tag{28}$$

Substituting $\alpha_i^c(\alpha_i^0) = (1 + \alpha_i^0)/3$ into the right-hand side of (28) we obtain

$$\frac{CS^c(\alpha_i^0, k) - v}{b} = \frac{8k\alpha_i^0 - 8k\left(\alpha_i^0\right)^2 - 11k + 18}{18}. \tag{29}$$

Using (29) and (17) we can express the difference between the consumer surpluses as

$$\frac{CS^c(\alpha_i^0, k) - CS(\alpha_i^0, k)}{b} = \frac{4k(1 - 3k)\alpha_i^0(\alpha_i^0 - 1) + 78k^2 - 107k + 36}{18\,(3k - 2)^2}. \tag{30}$$

One can easily see that the sign of Equation (30) is given by the sign of the numerator which we define by $\xi_1(\alpha_i^0, k)$. Let us also define $\xi_2(k) := 4k(1 - 3k)$. Note that $\xi_2(k)$ is positive, when $k < 1/3$, zero when $k = 1/3$ and negative otherwise. The discriminant of the function $\xi_1(\alpha_i^0, k)$ is given by $D = 12^2 k\,(3k - 1)\,(3k - 2)^2$. The discriminant is negative if $k < 1/3$, zero if $k = 1/3$, and positive otherwise. Hence, $\xi_2(k)$ is positive, while the discriminant is negative for $k < 1/3$, which implies that $\xi_1(\alpha_i^0, k)$ is positive for any α_i^0. Hence, consumer surplus is higher under compatibility than in the market sharing equilibrium under incompatibility in that region. If $k = 1/3$, then $\xi_1(\alpha_i^0, k) = 9$ for any α_i^0, and consumer surplus is again higher under compatibility. Consider now $k > 1/3$, for which the function $\xi_1(\alpha_i^0, k)$ has two roots, namely, $\alpha_1(k) = 1/2 +$

$(3/2)\left|3k-2\right|\sqrt{k(3k-1)}/(3k-1)$ and $\alpha_2(k) = 1/2 - (3/2)\left|3k-2\right|\sqrt{k(3k-1)}/(3k-1)$, it is straightforward that $\alpha_1(k) > \alpha_2(k)$ for any $k > 1/3$. The following claim shows how $\alpha_1(k)$ and $\alpha_2(k)$ are related to $\overline{\alpha}^0(k)$ and $\overline{\overline{\alpha}}^0(k)$.

Claim 4. *It holds that* $\alpha_1(k) > \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ *and* $\alpha_2(k) < \min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ *for any* $k > 1/3$.

Proof. We first show that $\max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\} = 1/2 + \left|3k-2\right|/2k$ and $\min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\} = 1/2 - \left|3k-2\right|/2k$. If $k < 2/3$, then $\max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\} = 1/k - 1$ and $1/2 + \left|3k-2\right|/2k = 1/2 - (3k-2)/2k = 1/k - 1$ and if $k > 2/3$, then $\max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\} = 2 - 1/k$ and $1/2 + \left|3k-2\right|/2k = 1/2 + (3k-2)/2k = 2 - 1/k$. The proof for $\min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\} = 1/2 - \left|3k-2\right|/2k$ proceeds in the same way. Consider now the difference $\alpha_1(k) - \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ which has the same sign as the expression $3\sqrt{k(3k-1)}/(3k-1) - 1/k$. The latter is positive if $(3k-1)(9k^3 - 3k + 1) > 0$ which is true if $9k^3 - 3k + 1 > 0$. Consider next the function $\xi_3(k) := 9k^3 - 3k + 1$. The derivative of the function $\xi_3(k)$ is given by $9k^2 - 1$, which is negative for $k < 1/3$ and positive for $k > 1/3$. Moreover, $\xi_3(k)$ reaches its local minimum at the point $k = 1/3$ with $\xi_3(1/3) = 1/3$. Hence, $\xi_3(k)$ is positive for any $k > 1/3$ and the difference $\alpha_1(k) - \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ is then also positive, what implies that $\alpha_1(k) > \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$. Consider now the difference $\alpha_2(k) - \min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ which has the sign opposite to the sign of the expression $3\sqrt{k(3k-1)}/(3k-1) - 1/k$. As we have shown that $3\sqrt{k(3k-1)}/(3k-1) - 1/k$ is positive for any $k > 1/3$, we can then conclude that $\alpha_2(k) < \min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ must hold. This completes the proof of the claim.

Since the roots of the function $\xi_1(\alpha_i^0, k)$ are such that $\alpha_1(k) > \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ and $\alpha_2(k) < \min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ and $4k(1-3k) < 0$ hold for $k > 1/3$, it follows that any α_i^0 for which the market sharing equilibrium under incompatibility emerges, $\xi_1(\alpha_i^0, k)$ takes only positive values. Hence, for any $k > 1/3$ consumers are better off under compatibility than in the market sharing equilibrium under incompatibility.

Case ii) Assume now that under incompatibility the monopoly equilibrium emerges with firm i gaining the monopoly position. Using (29) and the formula for consumer surplus under the monopoly equilibrium (which is given by $CS_i^M(\alpha_i^0, k) = v + (t/2)\left[1 - (\alpha_i^0)^2\right]$) we express the difference between the consumer surpluses as

$$\frac{CS^c(\alpha_i^0, k) - CS_i^M(\alpha_i^0, k)}{b} = \frac{k\left(\alpha_i^0\right)^2 + 8k\alpha_i^0 - 20k + 18}{18}. \tag{31}$$

The sign of the difference $CS^c(\alpha_i^0, k) - CS_i^M(\alpha_i^0, k)$ is given by the sign of the nominator which we define as $\xi_4(\alpha_i^0, k)$. The discriminant of the function $\xi_4(\alpha_i^0, k)$ is given by $D = 72k\,(2k - 1)$, which is negative for $k < 1/2$, zero if $k = 1/2$ and positive otherwise. Hence, as $k > 0$, then for $k < 1/2$ the function $\xi_4(\alpha_i^0, k)$ takes only positive values and consumers are better off under compatibility than in the monopoly equilibrium with firm i being the monopolist. Consider $k = 1/2$ for which $\xi_4(\alpha_i^0, 1/2) = \left(\alpha_i^0 + 4\right)^2/2$, which is positive for any α_i^0. Consider finally $k > 1/2$. The roots of the function $\xi_4(\alpha_i^0, k)$ are given by $\beta_1(k) := -4 + (3\sqrt{2k(2k-1)})/k$ and $\beta_2(k) := -4 - (3\sqrt{2k(2k-1)})/k$. It is straightforward that $\beta_2(k) < \beta_1(k)$ for any $k > 1/2$. We show that $\beta_1(k)$ is such that $\beta_1(k) < \overline{\alpha}^0(k)$. Solving $\beta_1(k) < \overline{\alpha}^0(k)$, we get $3\sqrt{2k(2k-1)} < 6k-1$, what can be simplified to $-6k < 1$ since $k > 1/6$. For any k the inequality $-6k < 1$ is true, hence $\beta_1(k) < \overline{\alpha}^0(k)$ follows. Since the roots of the function $\xi_4(\alpha_i^0, k)$ are such that $\beta_2(k) < \beta_1(k)$ and $\beta_1(k) < \overline{\alpha}^0(k)$ and $k > 0$, then for any α_i^0 for which the monopoly equilibrium with firm i gaining the monopoly position emerges ($\alpha_i^0 \geq \overline{\alpha}^0(k)$) the function $\xi_4(\alpha_i^0, k)$ takes only positive values and consumers are better off under compatibility than in the monopoly equilibrium with firm i being the monopolist.

Assume now that under incompatibility the monopoly equilibrium emerges with firm j gaining the monopoly position in which case consumer surplus is given by $CS_j^M(\alpha_i^0, k) = v + (t/2)\,[1 - (1-\alpha_i^0)^2]$. Note now that $CS^c(\alpha_i^0, k) = CS^c(1-\alpha_i^0, k)$ and $CS_j^M(\alpha_i^0, k) = CS_i^M(1-\alpha_i^0, k)$. As $CS^c(\alpha_i^0, k) > CS_i^M(\alpha_i^0, k)$ holds for any α_i^0, then because of symmetry consumers must also be better off for any α_i^0 if firm j ($j \neq i$) becomes the monopolist under incompatibility.

Q.E.D.

Proof of Proposition 9. *Case i).* We proceed by comparing social welfare under compatibility and incompatibility. Apart from the stand-alone value, v, under compatibility social welfare is given by the value of the network effects, b, and incurred switching costs, $(t/2)\left(\alpha_i^c - \alpha_i^0\right)(1 - 2\alpha_i^c)$. So that social welfare under compatibility can be (implicitly) expressed as

$$\frac{SW^c(\alpha_i^c, k) - v}{b} = 1 - \frac{k\left(\alpha_i^c - \alpha_i^0\right)(1 - 2\alpha_i^c)}{2}. \tag{32}$$

Substituting $\alpha_i^c(\alpha_i^0) = (1+\alpha_i^0)/3$ into (32) yields

$$\frac{SW^c(\alpha_i^0, k) - v}{b} = \frac{4k\alpha_i^0(1-\alpha_i^0) - k + 18}{18}. \tag{33}$$

Using (33) and (22) we can write the difference between social welfare under compatibility and social welfare in the market sharing equilibrium under incompatibility as

$$\frac{SW^c(\alpha_i^0, k) - SW^I(\alpha_i^0, k)}{b} = \frac{20k\alpha_i^0(\alpha_i^0 - 1)(1-3k) + 66k^2 - 103k + 36}{18\,(3k-2)^2}. \tag{34}$$

Define the numerator as $\varsigma_1(\alpha_i^0, k)$, which determines the sign of the right hand-side of (34). The discriminant of $\varsigma_1(\alpha_i^0, k)$ is given by $720k\,(3k-1)\,(3k-2)^2$, which is negative for $k < 1/3$, zero if $k = 1/3$ and positive otherwise. Note that $20k(1-3k)$ is positive if $k < 1/3$, zero if $k = 1/3$ and negative otherwise. Hence, for $k < 1/3$ the function $\varsigma_1(\alpha_i^0, k)$ takes only positive values for any α_i^0 and social welfare is higher under compatibility. If $k = 1/3$, then $\varsigma_1(\alpha_i^0, k) = 9$ and social welfare is again higher under compatibility. Consider next $k > 1/3$. The roots of the function $\varsigma_1(\alpha_i^0, k)$ are given by $\mu_1 := 1/2 + (3\,|3k-2|\,\sqrt{5k(3k-1)})/\,[10k(3k-1)]$ and $\mu_2 := 1/2 - (3\,|3k-2|\,\sqrt{5k(3k-1)})/\,[10k(3k-1)]$ with $\mu_2 = 1 - \mu_1$. In the following claim we describe the properties of those roots.

Claim 5. *The roots of the function $\varsigma_1(\alpha_i^0, k)$ have the following properties. If $1/3 < k < 5/6$, then $\mu_1 > \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ and $\mu_2 < \min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$. If $k = 5/6$, then $\mu_1 = \overline{\alpha}^0(k)$ and $\mu_2 = \overline{\overline{\alpha}}^0(k)$. If $5/6 < k \leq 1$, then $\mu_1 < \overline{\alpha}^0(k)$ and $\mu_2 > \overline{\overline{\alpha}}^0(k)$. If $1 < k < (103+\sqrt{1105})/132$, then $\mu_1 < 1$ and $\mu_2 > 0$. If $k = (103+\sqrt{1105})/132$, then $\mu_1 = 1$ and $\mu_2 = 0$. If $k > (103+\sqrt{1105})/132$, then $\mu_1 > 1$ and $\mu_2 < 0$.*

Proof. Recall that $\max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\} = 1/2 + |3k-2|\,/2k$ and $\min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\} = 1/2 - |3k-2|\,/2k$. Solving $\mu_1 > \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ for k we get $3\sqrt{5k(3k-1)} > 5(3k-1)$ for $k > 1/3$. The latter inequality can be simplified to $-6k > -5$ for $k > 1/3$, which is only true if $k < 5/6$, while for $k > 5/6$ the opposite holds. For $k = 5/6$ we get $\mu_1 = \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$. Solving $\mu_2 < \min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ for k we get $3\sqrt{5k(3k-1)} > 5(3k-1)$, what we showed to be true if $k < 5/6$, while for $k > 5/6$ the opposite holds. This proves the first part of the claim. Consider now $k > 1$, for which we have to know how μ_1 and μ_2 are related to 1 and 0, respectively. Solving $\mu_1 > 1$ we get $9(3k-2)^2 > 5k(3k-1)$, or equivalently, $9(3k-2)^2 > 5k(3k-1)$ which holds for $k > (103+\sqrt{1105})/132$, while for $k < (103+\sqrt{1105})/132$ the opposite is true and if

$k = (103 + \sqrt{1105})/132$, then $\mu_1 = 1$. Note that $(103 + \sqrt{1105})/132 > 1$. Solving $\mu_2 < 0$ is equivalent to solving $\mu_1 > 1$. This completes the proof of the claim.

We can now determine the sign of $\varsigma_1(\alpha_i^0, k)$. Consider first $1/3 < k < 5/6$. By Claim 5 we know that for $1/3 < k < 5/6$ μ_1 and μ_2 are such that $\mu_1 > \max\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$ and $\mu_2 < \min\{\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k)\}$. Hence, for any α_i^0 for which the market sharing equilibrium under incompatibility emerges $\varsigma_1(\alpha_i^0, k)$ takes only positive values as $20k(1-3k) < 0$ and social welfare is higher under compatibility. If $k = 5/6$, then $\varsigma_1(\alpha_i^0, k) = 0$ if $\alpha_i^0 = \overline{\alpha}^0(k)$ or if $\alpha_i^0 = \overline{\overline{\alpha}}^0(k)$ and positive for all other α_i^0 for which the market sharing equilibrium under incompatibility emerges. Consider now $5/6 < k \leq 1$ for which $\mu_1 < \overline{\alpha}^0(k)$ and $\mu_2 > \overline{\overline{\alpha}}^0(k)$ hold. Then $\varsigma_1(\alpha_i^0, k)$ is positive if $\alpha_i^0 \in (\mu_2, \mu_1)$, while $\varsigma_1(\alpha_i^0, k) = 0$ if $\alpha_i^0 = \mu_2$ or if $\alpha_i^0 = \mu_1$, and $\varsigma_1(\alpha_i^0, k)$ is negative if $\alpha_i^0 \in (\overline{\overline{\alpha}}^0(k), \mu_2)$ or if $\alpha_i^0 \in (\mu_1, \overline{\alpha}^0(k))$. Consider $k > (103 + \sqrt{1105})/132$ for which $\mu_1 > 1$ and $\mu_2 < 0$. Hence, for any α_i^0 it follows that $\varsigma_1(\alpha_i^0, k) > 0$. Consider now $k = (103 + \sqrt{1105})/132$ for which $\mu_1 = 1$ and $\mu_2 = 0$. Hence, $\varsigma_1(\alpha_i^0, k) > 0$ for any $\alpha_i^0 \notin \{0, 1\}$, and $\varsigma_1(\alpha_i^0, k) = 0$ for $\alpha_i^0 \in \{0, 1\}$. Consider finally $1 < k < (103 + \sqrt{1105})/(132)$ for which $\mu_1 < 1$ and and $\mu_2 > 0$. Then $\varsigma_1(\alpha_i^0, k)$ is positive, if $\alpha_i^0 \in (\mu_2, \mu_1)$, and $\varsigma_1(\alpha_i^0, k) = 0$ if $\alpha_i^0 = \mu_2$ or if $\alpha_i^0 = \mu_1$, while $\varsigma_1(\alpha_i^0, k)$ is negative if $\alpha_i^0 \in [0, \mu_2)$ or if $\alpha_i^0 \in (\mu_1, 1]$.

Case ii). Consider now the case that under incompatibility the monopoly equilibrium emerges. Using (23) and (33) we get the difference between social welfare under compatibility and under the monopoly equilibrium with firm i being the monopolist under incompatibility

$$\frac{SW^c(\alpha_i^0, k) - SW_i^M(\alpha_i^0, k)}{b} = \frac{k(\alpha_i^0 - 4/5)(\alpha_i^0 - 2)}{18}, \tag{35}$$

from which the result stated in the proposition follows immediately. *Q.E.D.*

Proof of Proposition 10. From (7) and the fact that in the market sharing equilibrium firms' prices are given by $p_i(\alpha_i^0, k) = kb\alpha_i^I(\alpha_i^0, k)$ we get firm's i profit in the market sharing equilibrium as

$$\pi_i(\alpha_i^0, k) = kb\left(\frac{k - 1 + k\alpha_i^0}{3k - 2}\right)^2. \tag{36}$$

Taking the derivative of (36) with respect to t we obtain

$$\frac{\partial \pi_i(\alpha_i^0, k)}{\partial t} = \frac{\partial \pi_i(\alpha_i^0, k)}{\partial k}\frac{\partial k}{\partial t} = \frac{(k - 1 + k\alpha_i^0)\left[3k\alpha_i^0(k - 2) + (3k^2 - 3k + 2)\right]}{(3k - 2)^3}. \tag{37}$$

Consider first all $k \neq 2$. Defining $\widetilde{\alpha}^0(k) := [3k(1-k)-2]/[3k(k-2)]$ and substituting $\widetilde{\alpha}^0(k)$ and $\overline{\overline{\alpha}}^0(k) = (1-k)k$ into the right-hand side of Equation (37) yields

$$\frac{\partial \pi_i(\alpha_i^0, k)}{\partial t} = \frac{3k^2(k-2)}{(3k-2)^3}\left[\alpha_i^0 - \overline{\overline{\alpha}}^0(k)\right]\left[\alpha_i^0 - \widetilde{\alpha}^0(k)\right]. \tag{38}$$

From Equation (38) we observe that the sign of $\partial \pi_i(\alpha_i^0, k)/\partial t$ is given by the sign of $[(k-2)/(3k-2)^3][\alpha_i^0 - \overline{\overline{\alpha}}^0(k)][\alpha_i^0 - \widetilde{\alpha}^0(k)]$. Let us now examine the properties of $\widetilde{\alpha}^0(k)$. Successive differentiation of $\widetilde{\alpha}^0(k)$ yields $\partial\widetilde{\alpha}^0(k)/\partial k = 3(k-2/3)(k+2)/\left[3k^2(k-2)^2\right]$ and $\partial^2\widetilde{\alpha}^0(k)/\partial k^2 = -2\left(3k^3+6k^2-12k+8\right)/[3k^3(k-2)^3]$. Note that $\partial\widetilde{\alpha}^0(k)/\partial k < 0$ if $k < 2/3$ and $\partial\widetilde{\alpha}^0(k)/\partial k > 0$ if $2/3 < k < 2$ and $k > 2$. Hence, $\widetilde{\alpha}^0(k)$ obtains a unique minimum at $k = 2/3$ with $\widetilde{\alpha}^0(2/3) = 1/2$. Solving $\widetilde{\alpha}^0(k) = 1$, we obtain $k_1 = (1/12)(9-\sqrt{33})$ and $k_2 = (1/12)(9+\sqrt{33})$ with $k_1 < 1/2$ and $k_2 < 4/3$. Taking the limit we obtain $\lim_{k\to\infty}\widetilde{\alpha}^0(k) = -1$. Hence, $\widetilde{\alpha}^0(k) \in (1/2, 1]$ if $k \in \{(1/12)(9-\sqrt{33}), 2/3) \cup (2/3, (1/12)(9+\sqrt{33})\}$ and for any other k it holds that either $\widetilde{\alpha}^0(k) < 0$ or $\widetilde{\alpha}^0(k) > 1$. In the intervals $k \in [1/2, 2/3)$ and $k \in (2/3, 1]$ the market sharing equilibrium only exist if $\alpha_i^0 \in (\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$ or $\alpha_i^0 \in (\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k))$ hold, respectively. We, therefore, have to analyze how $\widetilde{\alpha}^0(k)$ is related to $\overline{\alpha}^0(k)$ and $\overline{\overline{\alpha}}^0(k)$ in those intervals. The following claim shows that for $k \in [1/2, 2/3)$ it is true that $\widetilde{\alpha}^0(k) \in (\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$, while for $k \in (2/3, 1]$ it holds that $\widetilde{\alpha}^0(k) \in (\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k))$.

Claim 6. *It holds that* $\widetilde{\alpha}^0(k) - \overline{\alpha}^0(k) > 0$ *and* $\overline{\overline{\alpha}}^0(k) - \widetilde{\alpha}^0(k) > 0$ *for all* $k \in [1/2, 2/3)$, *while for all* $k \in (2/3, 1]$ *it holds that* $\overline{\alpha}^0(k) - \widetilde{\alpha}^0(k) > 0$ *and* $\widetilde{\alpha}^0(k) - \overline{\overline{\alpha}}^0(k) > 0$.

Proof. Simple calculations give $\overline{\overline{\alpha}}^0(k) - \widetilde{\alpha}^0(k) = 2(3k-2)/[3k(k-2)]$ which is strictly positive over the interval $k \in [1/2, 2/3)$ and negative over the interval $k \in (2/3, 1]$. Similarly, we obtain $\widetilde{\alpha}^0(k) - \overline{\alpha}^0(k) = 9[(4/3)-k][k-(2/3)]/[3k(k-2)]$ which is clearly strictly positive over the interval $k \in [1/2, 2/3)$. We know that k_1 and k_2 such that $k_1 < 1/2$ and $k_2 < 4/3$ solve $\widetilde{\alpha}^0(k) = 1$. Hence, it holds that $\widetilde{\alpha}^0(k) - \overline{\alpha}^0(k) < 0$ for $k \in (2/3, 1]$. This completes the proof of the claim.

Note, that for $k \in [1/2, 2/3)$ the market sharing equilibrium exists only if $\alpha_i^0 \in (\overline{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$. From Claim 6 we know that α_i^0 lies either in the interval $(\overline{\alpha}^0(k), \widetilde{\alpha}^0(k))$ or in the interval $(\widetilde{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$ for $k \in [1/2, 2/3)$. In the former case $\alpha_i^0 - \overline{\overline{\alpha}}^0(k) < 0$ and $\alpha_i^0 - \widetilde{\alpha}^0(k) < 0$, so that the right-hand side of Equation (38) is strictly negative as both $k-2<0$ and $3k-2<0$ hold. Hence, the firm's profit increases as switching costs increase if $\alpha_i^0 \in (\overline{\alpha}^0(k), \widetilde{\alpha}^0(k))$ for

$k \in (0, 2/3)$. Consider now the other case with $\alpha_i^0 \in (\widetilde{\alpha}^0(k), \overline{\overline{\alpha}}^0(k))$, where $\alpha_i^0 - \overline{\overline{\alpha}}^0(k) < 0$ and $\alpha_i^0 - \widetilde{\alpha}^0(k) > 0$, so that the the right-hand side of Equation (38) is strictly positive. Note now that for $k \in (2/3, 1]$ the market sharing equilibrium emerges only if $\alpha_i^0 \in (\overline{\overline{\alpha}}^0(k), \overline{\alpha}^0(k))$. From Claim 6 we know that α_i^0 lies either in the interval $(\overline{\overline{\alpha}}^0(k), \widetilde{\alpha}^0(k))$ or in the interval $(\widetilde{\alpha}^0(k), \overline{\alpha}^0(k))$ for $k \in (2/3, 1)$. Proceeding as before we get again that firm i's profit increases as switching costs increase if $\alpha_i^0 < \widetilde{\alpha}^0(k)$, whereas its profit decreases if $\alpha_i^0 > \widetilde{\alpha}^0(k)$ holds.

If $k = 2$, then the right-hand side of Equation (37) is given by $(1 + 2\alpha_i^0)/8 > 0$ for any α_i^0. *Q.E.D.*

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